①

Final: April 23 at 8:30 am
in LSK 201.

Workshops will run this week: review.

Last Webwork: #12 due Friday.

Grades: going into exam will be posted this/next week.

To study:
- Examples done in class.
- Webwork problems.
- Workshop problems.
- Practice Problems.
- Midterm 1 and 2 problems.
- Practice on past exams.
- MER Wiki.

Make sure you observe Course Outline and Learning Goals.

Exam Office Hours:

Mon, Tues, Wed. 20, 21, 22?

A) 11am B) 12, C) 1, D) 2, E) 3pm.

Mon: 11am - 1pm
Tues: 2pm - 4pm ← maybe review.
Wed: 12noon - 2pm.

②

Clickers: A) Ok
B) Lost.

Standard Q: Your ~~fri~~ friend finds the 2^{nd} degree taylor polynomial for a function $g(x)$ around $x = 1$, and uses it to approximate $g(2)$. Find a bound on the error in this approximation. if $g'''(x) = \frac{1}{x^2}$.

$$|R_2(x)| \le \frac{M\,|x-a|^3}{3!}$$

$$|g'''(x)| \le M \text{ on } [1,2].$$

$$|g'''(x)| \le |g'''(1)| = \frac{1}{1} = 1$$

So, $M = 1$.

$$|R_2(2)| \le \frac{1 \cdot |2-1|^3}{3!} = 1/6.$$

(3)

Harder Q: Now $g'''(x) = x(x-3)$.

Find M so that $|g'''(x)| \le M$ for $x \in [1,2]$.

Need abs. max/min.
Use closed interval method.

Critical points: $g'''(x) = x^2 - 3x$.

$$(g'''(x))' = 2x - 3.$$

$$0 = 2x - 3 \Rightarrow x = 3/2.$$

Check endpoints and critical points.

$$g'''(1) = 1 - 3 = -2$$
$$g'''(2) = \cancel{4} - 6 = -2.$$
$$g'''(3/2) = 9/4 - 9/2 = 9\left(\frac{1}{4} - 2/4\right)$$
$$= -9/4.$$
$$= -2\,1/4.$$

So, $M = 2\,1/4$.

$$|R_2(2)| \le \frac{2\,1/4}{3!}|2-1|^3.$$

④

Standard Q: Show that there is a c so that f is continuous

$$f(x) = \begin{cases} x^2, & x < c \\ x+1, & x \geq c \end{cases}.$$

x^2 and $x+1$ are continuous.

For continuity: $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$.

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^-} x^2 = c^2.$$

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^+} x+1 = c+1.$$

$$f(c) = c+1.$$

So, set $c^2 = c+1$.

$$c^2 - c - 1 = 0.$$

$$c = \frac{1 \pm \sqrt{1+4}}{2}.$$

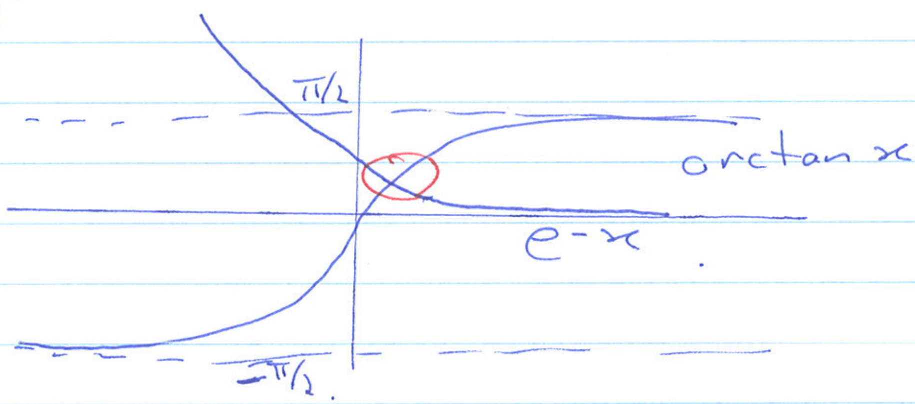
Harder Q: $f(x) = \begin{cases} \arctan x, & x < c \\ e^{-x}, & x \geq c \end{cases}$.

Want $\arctan c = e^{-c}$.

$g(c) = \arctan c - e^{-c} = 0$.

We need IVT. g is continuous.

$g(0) = 0 - e^{0} = -1 < 0$.



$g(\sqrt{3}) = \arctan(\sqrt{3}) - e^{-\sqrt{3}}$

$= \pi/3 - 1/e^{\sqrt{3}} > 0$.

x	$\tan x$
$\pi/6$	$1/\sqrt{3}$
$\pi/4$	1
$\pi/3$	$\sqrt{3}$

So there is some c that makes $g(c) = 0$.
ie. there is some c that makes $f(x)$ continuous.

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Standard:

Use the definition of the derivative to find $f'(x)$.

$$f(x) = \frac{x+1}{x-1}.$$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}.$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

$$f'(x) = \lim_{h \to 0} \frac{1}{h}\left(\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1} \right).$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{(x+h-1)(x-1)} \right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\cancel{(x+1)(x-1)} + h(x-1) - \cancel{(x+1)(x-1)} - h(x+1)}{(x+h-1)(x-1)} \right).$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{\cancel{h}(x-1) - \cancel{h}(x+1)}{(x+h-1)(x-1)}.$$

$$= \lim_{h \to 0} \frac{(x-1) - (x+1)}{(x+h-1)(x-1)}. = \lim_{h \to 0} \frac{-2}{(x+h-1)(x-1)}.$$

$$= \frac{-2}{(x-1)^2}.$$

Check with QR:

$$f'(x) = \frac{1(x-1) - 1(x+1)}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}.$$

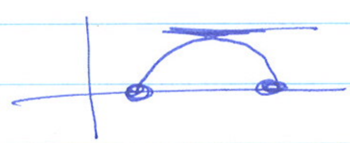
Standard Q: Show that on the interval $(-\infty, -1]$ $f(x) = x^3 + x^2 + 1$ has exactly one root.

f - continuous and differentiable everywhere.

$f(-2) = -8 + 4 + 1 = -3 < 0$
$f(-1) = -1 + 1 + 1 = 1 > 0.$

$\Rightarrow$ there exists c in $(-2, -1)$ with $f(c) = 0$.
$\Rightarrow$ at least one root.

If we had 2 roots: then by Rolle's Theorem $f'(x) = 0$ somewhere.



We want to show $f'(x) \neq 0$ on $(-\infty, -1]$.

$$f'(x) = 3x^2 + 2x$$
$$= x(3x + 2).$$

Critical points: $x = 0, -2/3$.

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So, we can't have two roots on $(-\infty, -1]$ since $f'(x) \neq 0$ on $(-\infty, -1]$.

(this comes from Rolle's Theorem).

Harder Q: Show that $f(x)$ has exactly one root on $(-\infty, \infty)$.

So, no root on $(-1, \infty)$.

	$(-\infty, -2/3)$	$(-2/3, 0)$	$(0, \infty)$
$f(x)$	$\nearrow$	$\searrow$	$\nearrow$
$f'(x)$	>0	<0	>0

local min.

$f(-1) = 1 > 0$
$f(0) = 1 > 0$.