① Jan 13

Office Hours: Wed. 9:30 - 11 am } LSK 300C
Thurs. 1 - 2:30 pm }
or by appointment in my office
AA 137.

Workshops: This week!

Webwork: Assignment 0 ← no marks.
Homework 1 diagnostic ← for marks.
Assignment 1 ← for marks

Precalc: Connect

Practice Problems: Do them

§2.5 Continuity

Last class we saw that for polynomials

$$\lim_{x \to a} f(x) = f(a) \quad \left(\text{direct substitution}\right)$$

Today we talk about all functions with this property.

② Jan 13

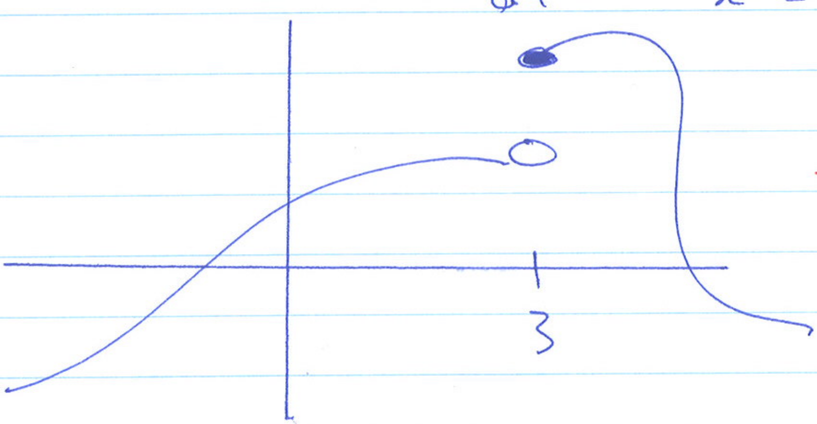
Definition: A function is continuous at $x = a$ if

$$\lim_{x \to a} f(x) = f(a).$$

A function can fail to be continuous (be discontinuous) at $x = a$ in 3 ways:

- $f(a)$ D.N.E.
- $\lim_{x \to a} f(x)$ D.N.E.
- $\lim_{x \to a} f(x) \neq f(a)$.

Clicker Q1: Why is this function discontinuous at $x = 3$.



A: $f(3)$ DNE

→ B: $\lim_{x \to 3} f(x)$ DNE

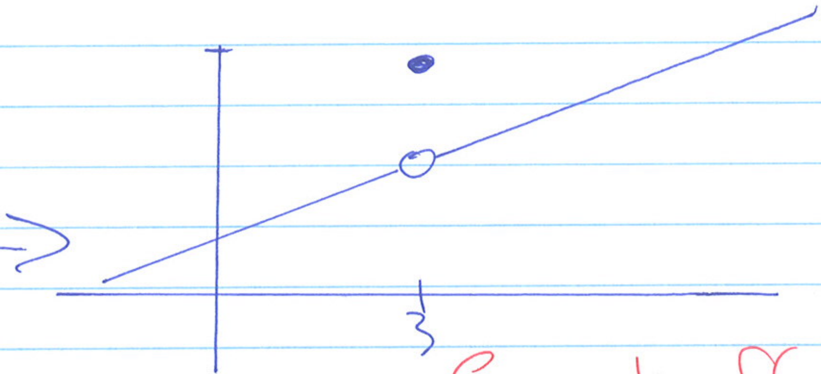
C: They exist but are not equal.

$f(3)$ and $\lim_{x \to 3} f(x)$.

"jump discontinuity"

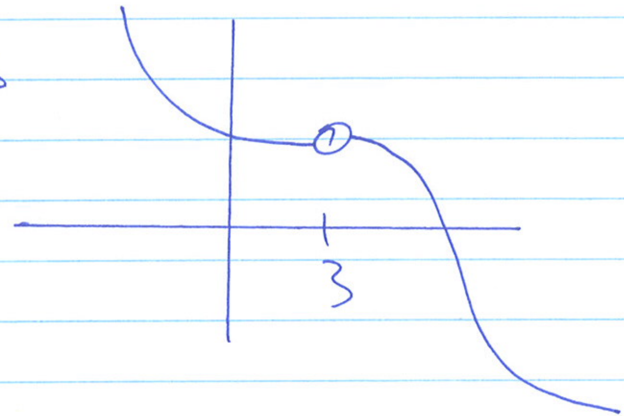
③ Jan 13

Clicker Q2:



C - $\lim_{x \to 3} f(x) \neq f(3)$

An example of A is



removable discontinuity

The function in Clicker Q1 is continuous from the right

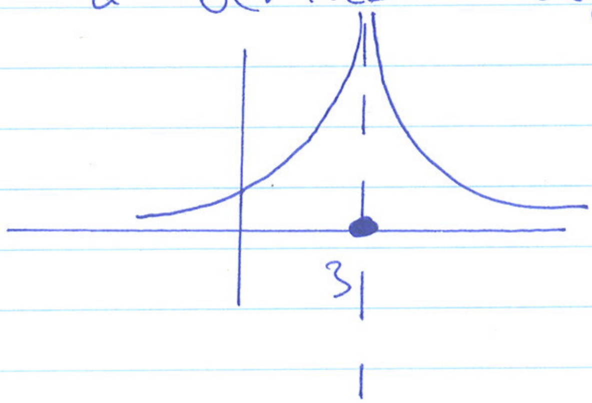
i.e. $\lim_{x \to 3^+} f(x) = f(3)$

Similarly a function is continuous from the left if

$$\lim_{x \to a^-} f(x) = f(a).$$

(4) Jan 13

You will also have a discontinuity at a vertical asymptote



$$\lim_{x \to 3} f(x) = \infty .$$

'infinite discontinuity'

Example: Is $f(x) = |x-1|$ continuous at $x = 1$?

We want to check if

$$\lim_{x \to 1} f(x) = f(1) .$$

Compute $f(1) = 0$.

Compute $\lim_{x \to 1} |x-1|$

So, $|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}.$

$$\left(|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}. \right)$$

(5) Jan 13

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} |x-1| = \lim_{x\to 1^-} -(x-1)$$

$$= -(1-1) = 0.$$

$$\lim_{x\to 1^+} |x-1| = \lim_{x\to 1^+} (x-1) = 1-1 = 0.$$

So, $\lim_{x\to 1} f(x) = 0 = f(1)$

So f is continuous at ~~0~~. $x = 1$.

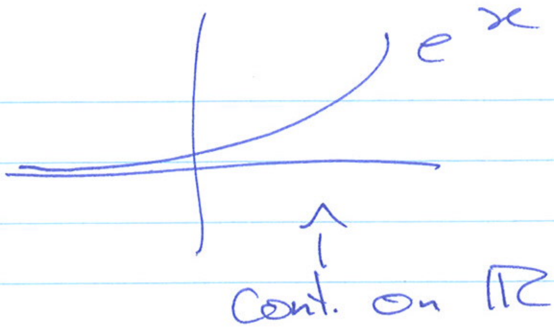
Definition: A function is continuous on an interval if it is continuous at each point in that interval.

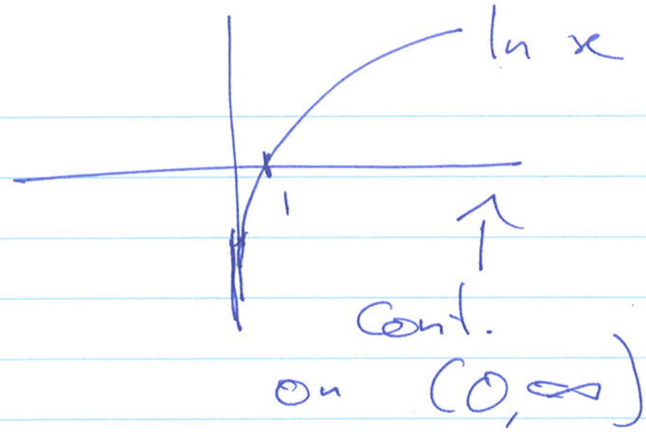
Which functions are continuous?

Theorem: The following functions are continuous on their domains.

- polynomials
- rational
- roots/powers
- trig/trig inverses (later)
- exponential
- logarithms

⑥ Jan 13





Additionally, combinations of continuous functions are continuous (kind of like limit laws)

<u>Theorem</u>: If f and g are continuous then

constant

$$f+g, \quad f-g, \quad cf$$

$$f \cdot g, \quad f/g \quad (g \neq 0)$$

are continuous.

<u>Example</u>: When is $\dfrac{e^x + \cos x}{\sin x}$ continuous?

(guess: $x \neq k\pi$, $k = 0, \pm 1, \pm 2, \ldots$)

The only problem is when $\sin x = 0$.

Continuous everywhere except when $x = k\pi$, $k = 0, \pm 1, \pm 2, \ldots$

Compositions of continuous functions are also continuous.

That is if g is continuous at a
f is continuous at $g(a)$

then $(f \circ g)(x) = f(g(x))$
is continuous at a.

Examples $\ln((x-1)^2)$.

$(x-1)^2$ - continuous $\ln x$ is continuous on $(0, \infty)$.

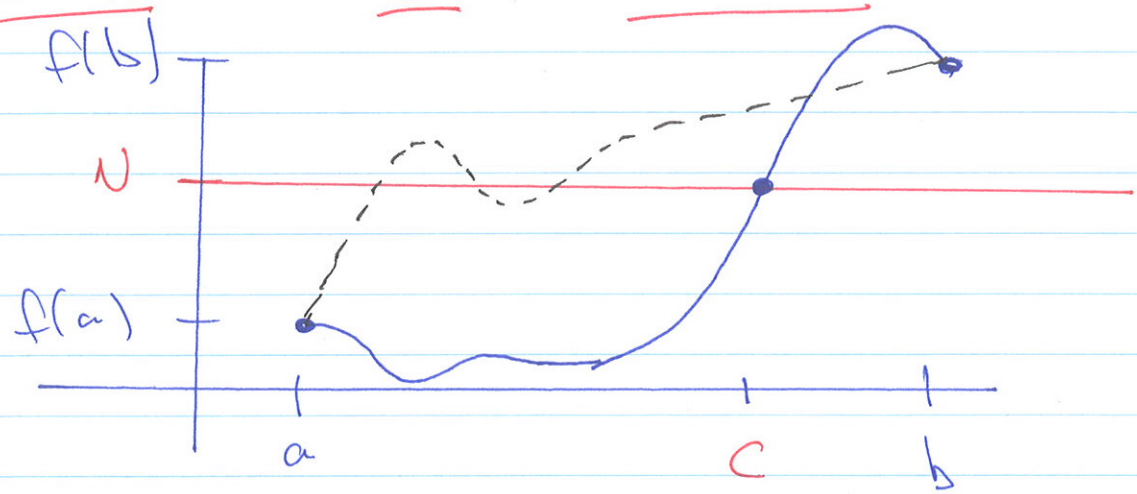
So we need $(x-1)^2 > 0$.

We know $(x-1)^2 \geq 0$.
We want to find where $(x-1)^2 = 0$
~~We therefore only need when $(x-1)^2 \neq 0$~~
or $x = 1$.

So $\ln((x-1)^2)$ continuous everywhere except at $x = 1$.

(8) Jan 13

Intermediate Value Theorem

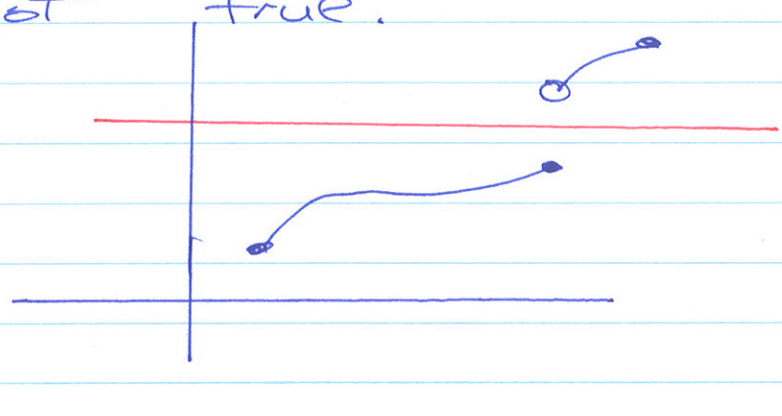


Theorem: (IVT)

Suppose f is continuous on $[a,b]$. Let N be any number between $f(a)$ and $f(b)$. Then there exists a c between a and b where

$$f(c) = N.$$

Remark: If the function can jump (discontinuous) the theorem is not true.

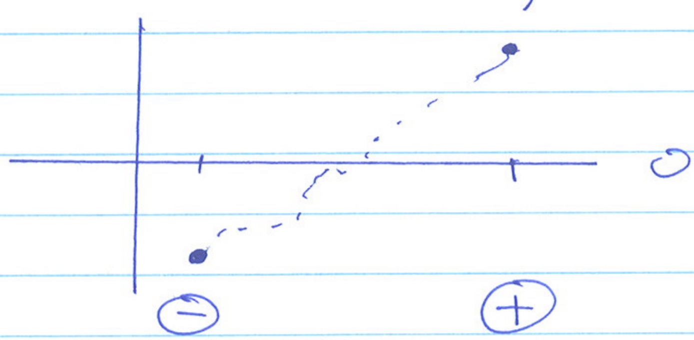


(9) Jan 13

Example: Show that $e^{-x} - 2(x+1) = 0$ has a solution.

let $f(x) = e^{-x} - 2(x+1)$

The idea is to find a point where $f(x) < 0$ and where $f(x) > 0$ then apply IVT.



Solution: First of all, $f(x)$ is continuous

Now let's guess some points.

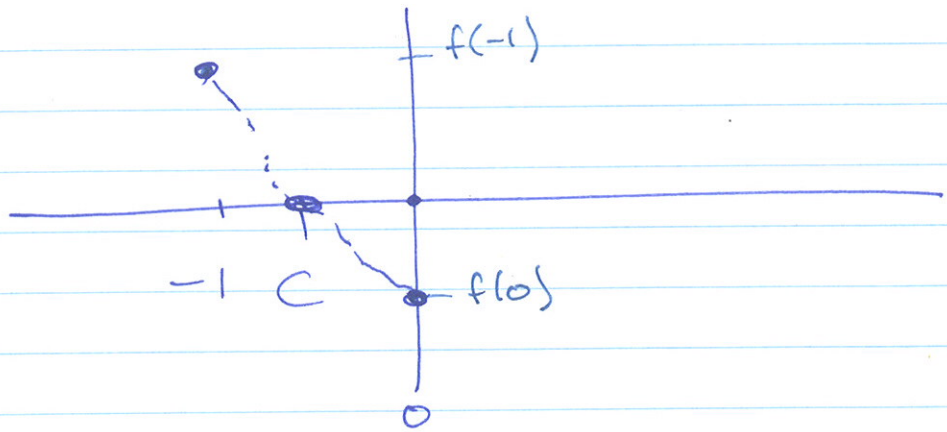
$$f(0) = e^{0} - 2 = 1 - 2 = -1 < 0.$$

$$f(-1) = e^{+1} - 2(-1+1)$$
$$= e^{+1} > 0.$$

So by the IVT there is a point c between -1 and 0 where

$$f(c) = 0.$$

⑩ Jan 13



We cannot find the c but we know it exists.