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Math 180 Section 201

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Course Webpage:

www.math.ubc.ca/~colesmp/teaching/M180.html

Book: Stewart, 7th ed.

Evaluation: Final Exam: 50%
Midterms (2): 16% each.
Webwork: 9%
next week → Workshops: 7%
Clickers: 2%.

Webwork: Homework 0 - not for marks.
Diagnostic Test - for marks.
- 1 h.

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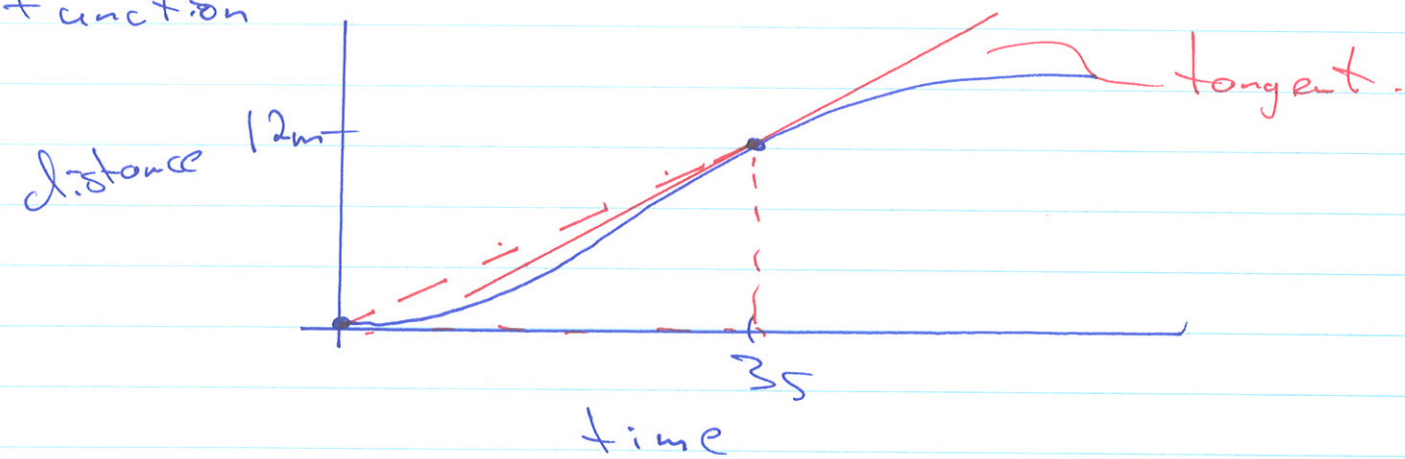
§2.1 (page 82) Tangents / Velocity

You are driving in your car and your distance from where you started is given by the following function



We can find average velocity over the first 3s.

$$V_{ave} = \frac{\text{change in distance}}{\text{change in time}}$$

$$= 12m/3s = 4m/s.$$

using $m = \frac{rise}{run}$.

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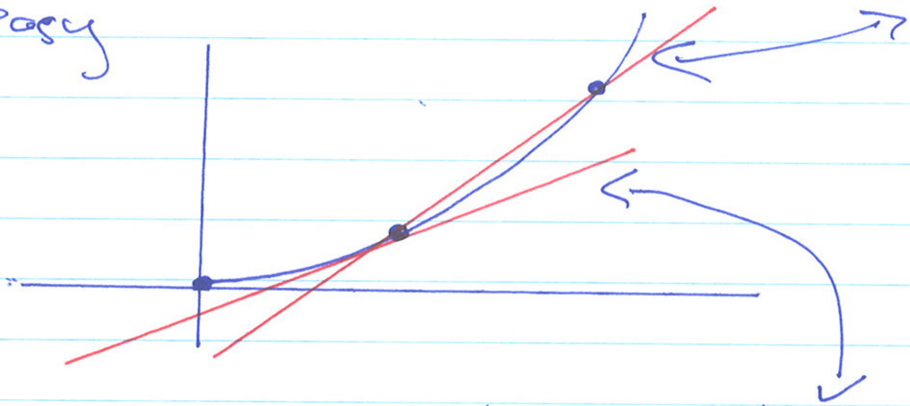
To find instantaneous velocity (how fast are you going at 3s) we need to find the slope of the tangent line

Finding the slope of a secant line is easy



Finding the slope of the tangent line is harder.

To find the tangent line we need limits.

Idea! Use secant lines to approximate the slope of the tangent line.

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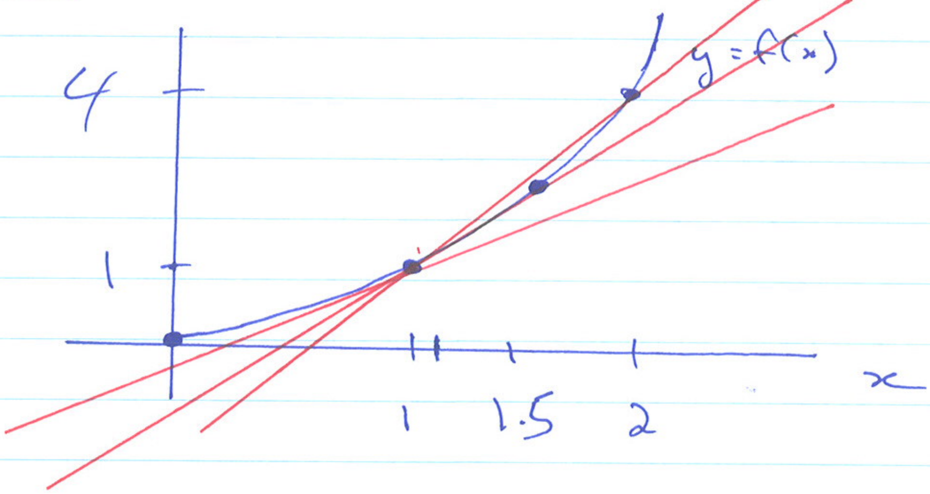
Example: Consider $f(x) = x^2$.



slope of secant between $x=1$, $x=2$

$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2)^2 - (1)^2}{2 - 1} = 3.$$

$x = 1, 1.5$, $m_{sec} = \frac{(1.5)^2 - 1^2}{1.5 - 1} = 2.5$

$x = 1, 1.1$, $m_{sec} = \frac{(1.1)^2 - 1^2}{1.1 - 1} = 2.1$

$x = 1, 1.01$ $m_{sec} = \frac{(1.01)^2 - 1^2}{1.01 - 1} = 2.01$

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It looks like the slope of the tangent line is about 2.

The way to know for sure is to use limits:

Generally, for x between 1 and $1 + h$ ← really small.

$$m_{sec} = \frac{(1+h)^2 - 1^2}{(1+h) - 1} = \frac{(1+h)^2 - 1}{h}.$$

In other words what we want to know is

$$\lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = ? \quad (= 2)$$

§ 2.2 Limits (page 87)

We write $\lim_{x \to a} f(x) = L$

to mean the limit of $f(x)$ as x approaches a is L.

We might also write $f(x) \to L$ as $x \to a$.

Example: Find $\lim_{x \to 1} f(x)$ where

$$f(x) = \begin{cases} x+1, & x \neq 1 \\ 3, & x = 1 \end{cases}.$$

To get an idea of what is going on let's try some values of x close to 1.

$f(1.1) = 2.1$ $\qquad$ $f(0.9) = 1.9$

$f(1.01) = 2.01$ $\qquad$ $f(0.99) = 1.99$.

It looks like 2 is a good guess. (is correct).

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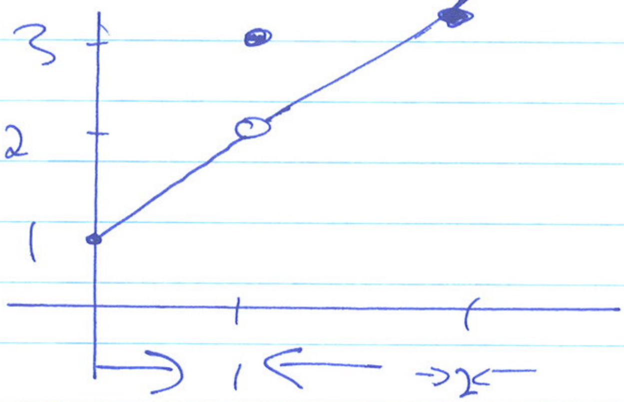
Note! $f(1) = 3 \neq 2$.

It doesn't matter what the value of $f(1)$ is, we just care about $f(x)$ when x is close to 1.

We can plot this function.



So, $\lim_{x \to 1} f(x) = 2$.

$\lim_{x \to 2} f(x) = ?$

$= 3.$

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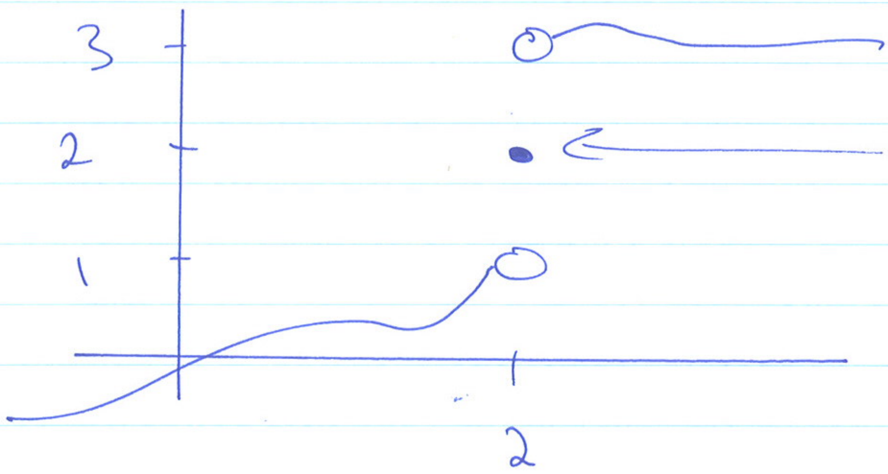
Question: what is $\lim_{x \to 2} f(x)$ for



$f(2) = 2$.

A: 1
B: 2
C: 3
D: both 1 and 3
E: none of the above.

This limit doesn't exist.

There is no single number that $f(x)$ approaches.

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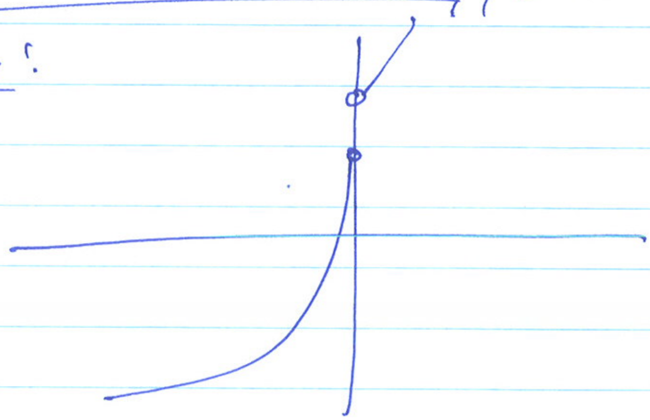
This function does have one sided limits.

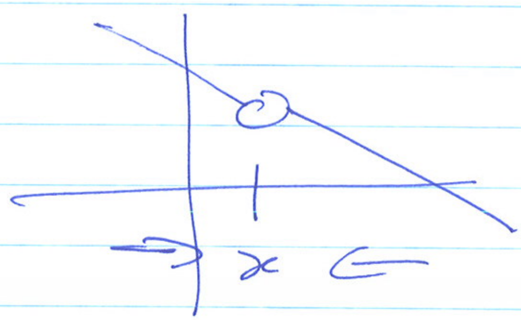
$$\lim_{x \to 2^-} f(x) = 1 \quad , \quad \lim_{x \to 2^+} f(x) = 3$$

one sided limit from below

one sided limit from above.

assside!





Note! (p. 92)

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$.

Example: Both of the one sided limits in this function do not exist.

$$\lim_{x \to 0} \sin(1/x)$$

It oscillates wildly near zero.
It does not approach a single value.
Neither one sided limit exists.
The full limit does not exist.

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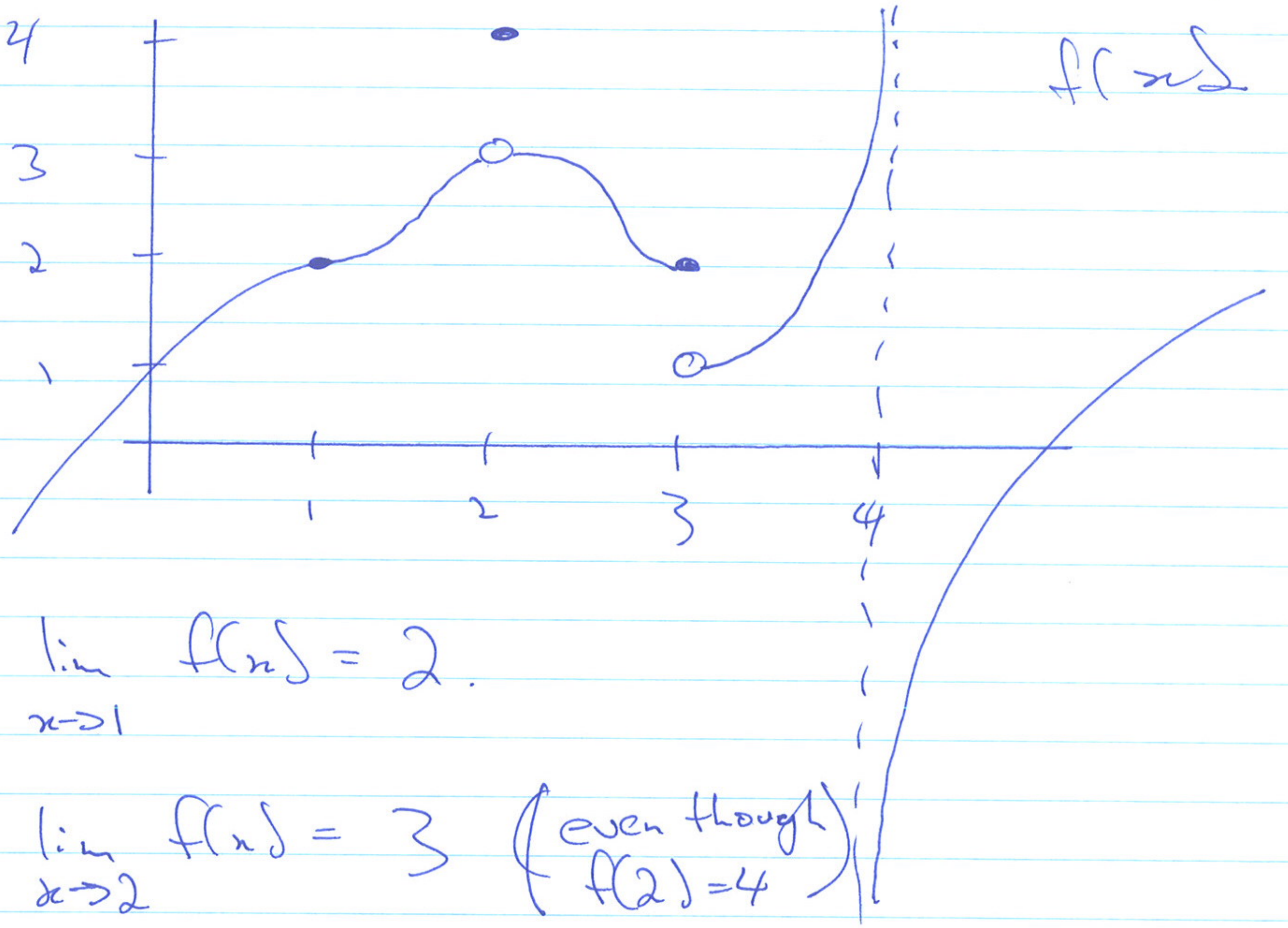
Ex: Find the one sided limits and full limit of f(x) as $x \to 1$, $x \to 2$, $x \to 3$, $x \to 4$.



$$\lim_{x \to 1} f(x) = 2.$$

$$\lim_{x \to 2} f(x) = 3 \quad \left(\begin{array}{c}\text{even though} \\ f(2) = 4\end{array}\right)$$

$$\lim_{x \to 3^-} f(x) = 2 \quad , \quad \lim_{x \to 3^+} f(x) = 1$$

$\lim_{x \to 3} f(x)$ does not exist.

What about $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$.

We write $\lim_{x\to 4^-} f(x) = \infty$

to means $f(x)$ gets really big as $x \to 4^-$

also, $\lim_{x\to 4^+} f(x) = -\infty$

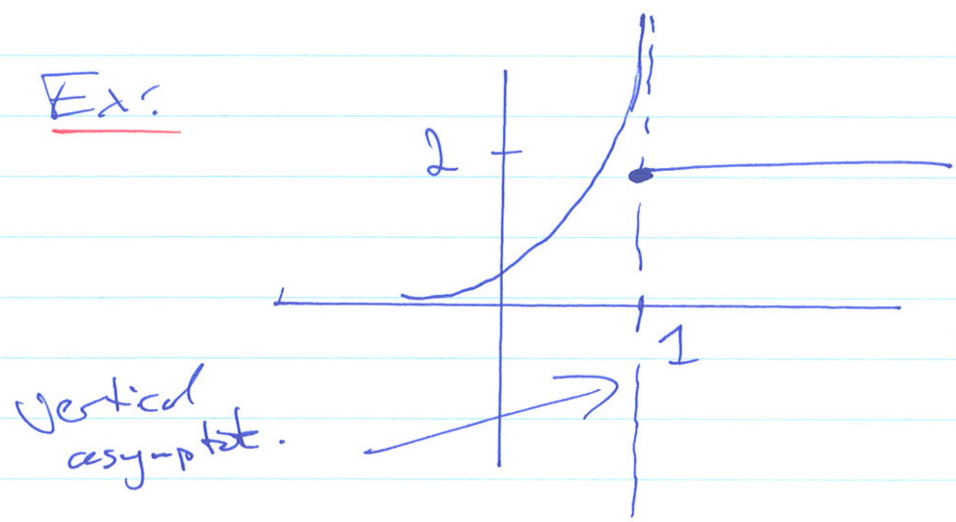
means $f(x)$ gets really big (but negative) as $x \to 4^+$.

Note: ∞ and $-\infty$ are not numbers.

Remark: If $\lim_{x\to a^\pm} f(x) = \pm\infty$

we say f has a vertical asymptote at a.

Ex:



$\lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to 1^-} f(x) = \infty$.