① Jan. 8.

Website:

www.math.ubc.ca/~colesmp/teaching/M180.html

Email: colesmp@math.ubc.ca

Workshops: Next Week

Webworks:

not for marks.

"Assignment 0" is open due Thurs. 15

"Homework 1 diagnostic" open due Thurs. 15

for marks

— timed. 1 hour from when you start.

"Assignment 1" to open. due Friday 16

— first regular assignment.

Clickers: Start today

Last class: Tangents/Velocity/Limits.

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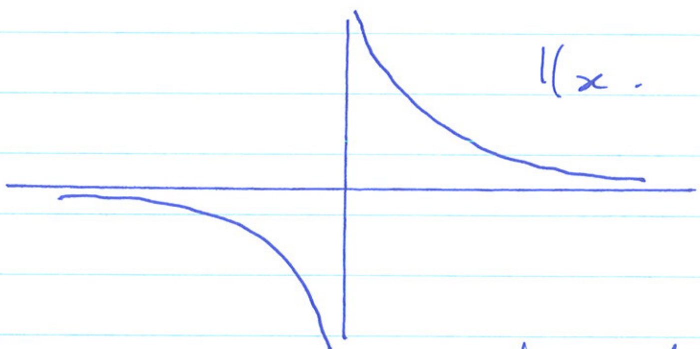
Office Hours: (tentative)

Wed: 9:30 - 11am
Thurs: 1 - 2:30pm } LSK 300C.

or by appointment.

Clicker Q: What is $\lim_{x \to 0} \frac{1}{x}$ ?

A: $\infty$
B: $-\infty$
→ C: The limit does not exist.




$\lim_{x \to 0^+} 1/x = \infty$

above/right

$\lim_{x \to 0^-} 1/x = -\infty$

below/left.

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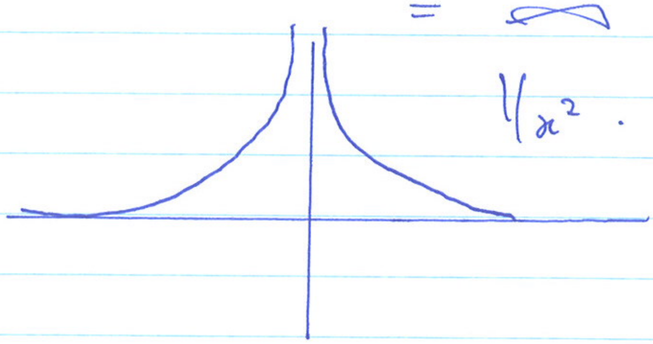
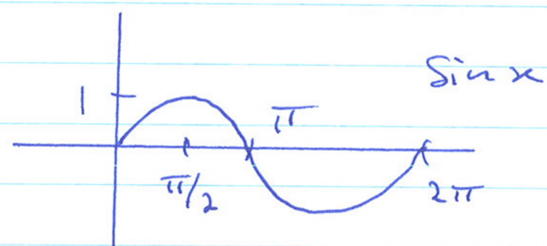
What about $\lim_{x \to 0} \frac{1}{x^2}$ ?

$= \infty$

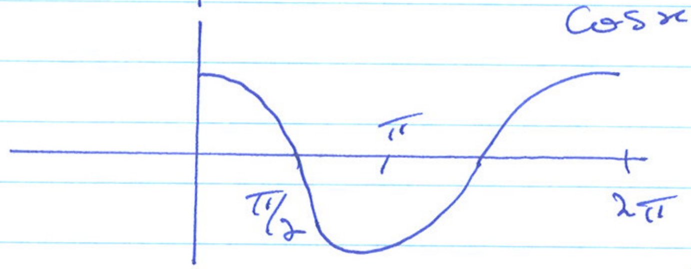



Examples What is $\lim_{x \to \pi/2^-} \tan x$ ?

Well $\tan x = \frac{\sin x}{\cos x}$




$x = \pi/2$ is a candidate for a vertical asymptote since $\cos(\pi/2) = 0$.




$\lim_{x \to \pi/2^-} \tan x = \infty$.

So, $x = \pi/2$ is a vertical asymptote.

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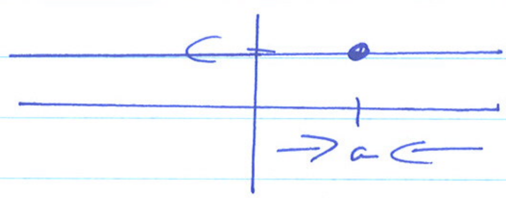
# § 2.3 Limit Laws / Computing Limits

Clicker Q: Let $c$ be a constant. What is

$$\lim_{x \to a} c = ?$$

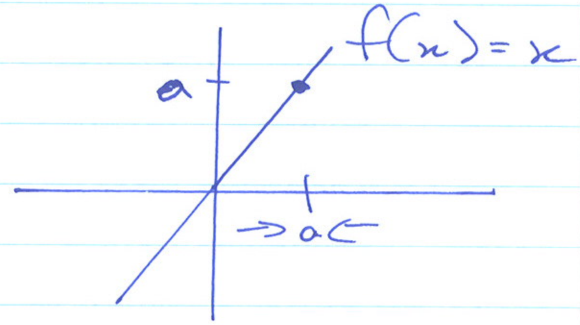
A = $a$
B = $0$
C = $c$
D = $1$.




$f(x) = c$.

Clicker Q: What is $\lim_{x \to a} x = ?$

A = $a$
B = $0$
C = $x$
D = $7$



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## § 2.3 Limit Laws / Computing Limits

Let $c$ be a constant and suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

(this table is on p. 99–100 in text).

1. $\lim_{x\to a}\left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

2. $\lim_{x\to a}\left(f(x)-g(x)\right) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

3. $\lim_{x\to a}\left(c\,f(x)\right) = c\lim_{x\to a} f(x)$

4. $\lim_{x\to a}\left(f(x)\,g(x)\right) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$.

(The proofs of these require § 2.4).

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Example:

$$\lim_{x \to 0} (3x + 6).$$

Well, $\lim_{x \to 0} (3x+6) = \lim_{x \to 0} 3x + \lim_{x \to 0} 6.$

(rule 1).

$$= 3 \lim_{x \to 0} x + \lim_{x \to 0} 6$$

(rule 3).

$$= 3 \cdot 0 + 6 = 6.$$

Example: $\lim_{x \to 4} \frac{x^2}{1+x} = \frac{\lim_{x \to 4} x^2}{\lim_{x \to 4} (1+x)}$

(rule 5)

we have to watch the bottom for "0".

$$= \frac{\lim_{x \to 4} (x \cdot x)}{\lim_{x \to 4} 1 + \lim_{x \to 4} x}$$

(rule 7)

$$= \frac{(\lim_{x \to 4} x)(\lim_{x \to 4} x)}{1 + 4}.$$

$$= \frac{4 \cdot 4}{5} = 16/5 .$$

(note no 0 in denominator).

Wait a minute... all we did was plug in $x = 4$.

Theorem: (Direct Substitution Property, p. 101)

If $f$ is a polynomial or a rational function and $a$ is in the domain of $f$ then,

$$\lim_{x \to a} f(x) = f(a) .$$

So,
$$\lim_{x \to 0} (3x+6) = 3 \cdot 0 + 6 = 6 .$$

$$\lim_{x \to 4} \frac{x^2}{1+x} = \frac{4^2}{1+4} = 16/5 .$$

Example: Find $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x - 2}$.

$$= \lim_{x \to 2} \frac{(x-2)(x-3)}{(x-2)}.$$

$$= \lim_{x \to 2} x - 3 = 2 - 3 = -1.$$

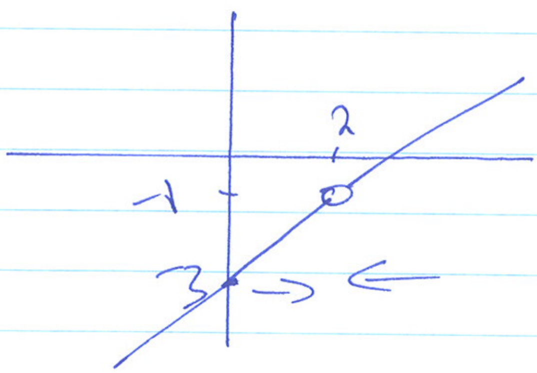
If we substituted $x = 2$ right away

$$f(2) = \frac{2^2 - 5 \cdot 2 + 6}{x - 2} = \text{"}\frac{0}{0}\text{"}.$$

$f(2)$ is undefined.

In other-words.

$$f(x) = \begin{cases} x - 3 & x \neq 2. \\ \text{undefined}, & x = 2. \end{cases}$$



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When you get "$\frac{0}{0}$" you need to do something.

Algebra / Cancellation / Substitute.

Example: Last class we were wondering about

$$\lim_{h \to 0} \frac{(1+h)^2 - 1}{h} = \text{"}\frac{0}{0}\text{"}.$$

We plugged in very small h values and got numbers close to 2.

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}.$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h} = \lim_{h \to 0} \frac{h(2+h)}{h}.$$

$$= \lim_{h \to 0} 2 + h = 2.$$

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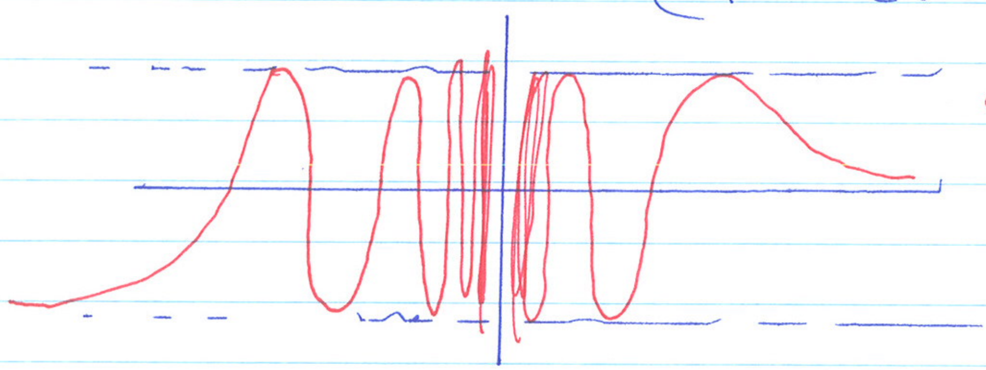
<u>Example</u>: Let $f(x) = x^2 \sin(1/x)$.

Find $\lim_{x \to 0} x^2 \cdot \sin(1/x)$.

It is tempting to write

$$\cancel{=} \underbrace{\left(\lim_{x\to 0} x^2\right)}_{=\,0} \cdot \left(\lim_{x \to 0} \sin(1/x)\right) = 0.$$

this limit does not exist. (last class)



$\sin(1/x)$

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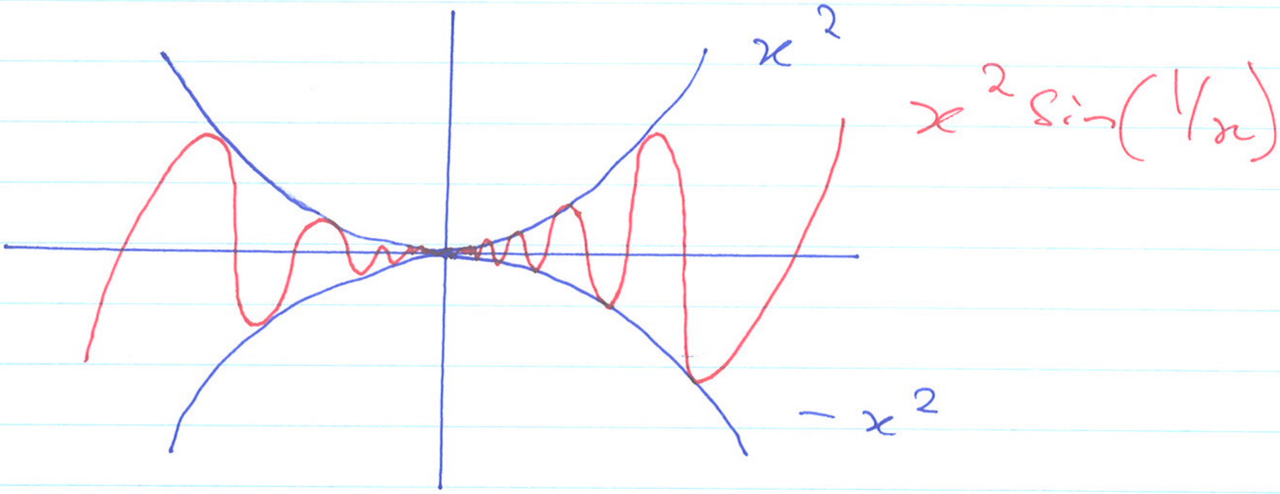
Notice : $-1 \le \sin(1/x) \le 1$




The $x^2$ is squeezing the $\sin(1/x)$.

Theorem: (Squeeze Theorem) (p.105)

Take 3 functions, f, g, h and suppose

$$g(x) \le f(x) \le h(x)$$

except possibly at $x = a$.

If $\lim_{x \to a} g(x) = L$ and $\lim_{x \to a} h(x) = L$

Then $\lim_{x \to a} f(x) = L$.

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Now we can solve the problem.

Solution: We want to find

$$\lim_{x \to 0} x^2 \sin(1/x).$$

Observe that

$$-1 \le \sin(1/x) \le 1$$

and therefore we also have (mult. by $x^2$)

$$-x^2 \le x^2 \sin(1/x) \le x^2.$$

Note that $\lim_{x \to 0} (-x^2) = \lim_{x \to 0} x^2 = 0.$

So, by the squeeze theorem

$$\lim_{x \to 0} x^2 \sin(1/x) = 0.$$

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# More Limit Problems

(try some or all of these)

- $\lim_{x \to 0} |x|$
- $\lim_{x \to 1} \frac{|x-1|}{x-1}$
- $\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x}$
- $\lim_{x \to -5} \frac{\frac{1}{x} + \frac{1}{5}}{5+x}$
- $\lim_{x \to 2} f(x)$ where $3x \le f(x) \le x^3 - 3x + 4$.