① March 17.

Is there interest in me holding a go over midterm session?

Next Week. Wednesday 5-7 pm.

§4.3 How Derivatives Affect the Shape of a Graph

Increasing/Decreasing

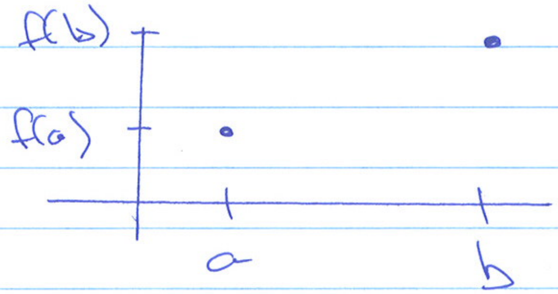
(a) If $f'(x) > 0$ (on some interval) then $f(x)$ is increasing (on that interval).

(b) If $f'(x) < 0$ (on some interval) then $f(x)$ is decreasing (on that interval).

Proving these statements is an application of the Mean Value Theorem.

We prove (a).

Suppose that $f'(x) > 0$.
We want to show that $f(x)$ is increasing.



②

Take a and b with $a < b$
$f(x)$ will be increasing if $f(a) < f(b)$.
So, we show $f(a) < f(b)$.

Now MVT ~~says~~: gives:

$$f'(c) = \frac{f(b) - f(a)}{(b - a)}.$$

for some c between a and b.

or $f(b) - f(a) = \underbrace{(b-a)}_{>0} \underbrace{f'(c)}_{>0}$.

So $f(b) - f(a) > 0$
So, $f(b) > f(a)$. Done $\square$.

(b) is similar.

Example: $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

Find the critical points.
Find where f is inc./dec.
Identify all local max/min.

(3)

$$f'(x) = 12x^3 - 12x^2 - 24x.$$

Set $f'(x) = 0$.

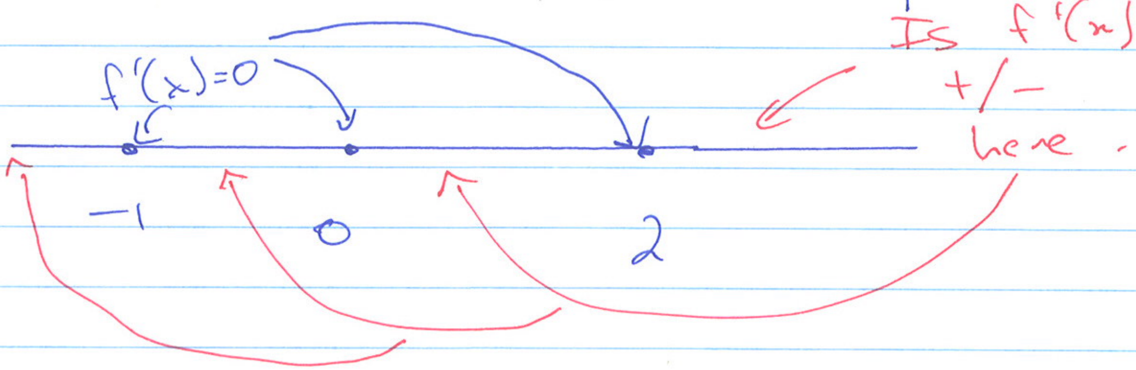
$$\begin{aligned} 0 &= 12x^3 - 12x^2 - 24x. \\ &= 12x(x^2 - x - 2) \\ &= 12x(x-2)(x+1). \end{aligned}$$

Critical points: $x = 0, 2, -1$.
(Note $f'(x)$ exists everywhere).

To find where f is inc./dec. we find where $f'(x)$ is positive/negative.

local min   local max   local min

	$(-\infty, -1)$	$(-1, 0)$	$(0, 2)$	$(2, \infty)$
$f(x)$	↘	↗	↘	↗
$f'(x)$	< 0	> 0	< 0	> 0



(4)

$$f'(x) = 12x(x-2)(x+1).$$

If $x \in (-\infty, -1)$ then $12x < 0$.
$x-2 < 0$.
$x+1 < 0$.

$$f'(x) = \underset{(-)}{12x}\,\underset{(-)}{(x-2)}\,\underset{(-)}{(x+1)}$$
$$= (-).$$
$$f'(x) < 0$$

Clicker Q: On $x \in (-1, 0)$ → A: $f'(x) > 0$
B: $f'(x) < 0$.

$$f'(x) = \underset{(-)}{12x}\,\underset{(-)}{(x-2)}\,\underset{(+)}{(x+1)}$$
$$f'(x) > 0.$$

On: $(0, 2)$. Clicker Q A: $f'(x) > 0$
→ B: $f'(x) < 0$

$$f'(x) = (+)(-)(+)$$
$$= (-)$$

On: $(2, \infty)$. $f'(x) = (+)(+)(+)$
$= (+)$.

f inc. on $(-1, 0) \cup (2, \infty)$

union

f dec. on $(-\infty, -1) \cup (0, 2)$

At $x = -1$: min
$x = 0$: max
$x = 2$: min.

This process is called the First Derivative Test.

(5)

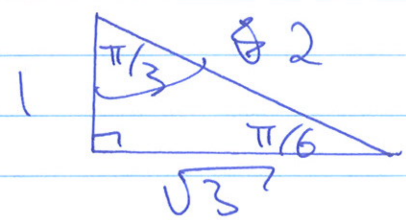
Example: $f(x) = x + 2\sin x$
on $0 \le x \le 2\pi$.

$f'(x) = 1 + 2\cos x$.

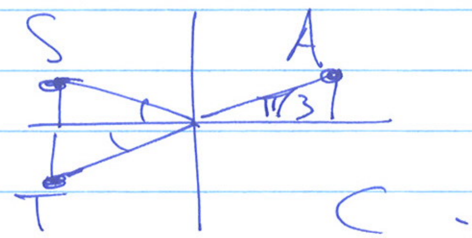
Critical Points: $0 = 1 + 2\cos x$

$-\frac{1}{2} = \cos x$.

SOHCAHTOA.



$\cos(\pi/3) = 1/2$.



$\pi - \pi/3 = 2\pi/3$

$\pi + \pi/3 = 4\pi/3$.

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$.

	$(0, \frac{2\pi}{3})$	$(\frac{2\pi}{3}, \frac{4\pi}{3})$	$(\frac{4\pi}{3}, 2\pi)$
$f(x)$	$\nearrow$	$\searrow$	$\nearrow$
$f'(x)$	>0	<0	>0

local max　　　　local min.

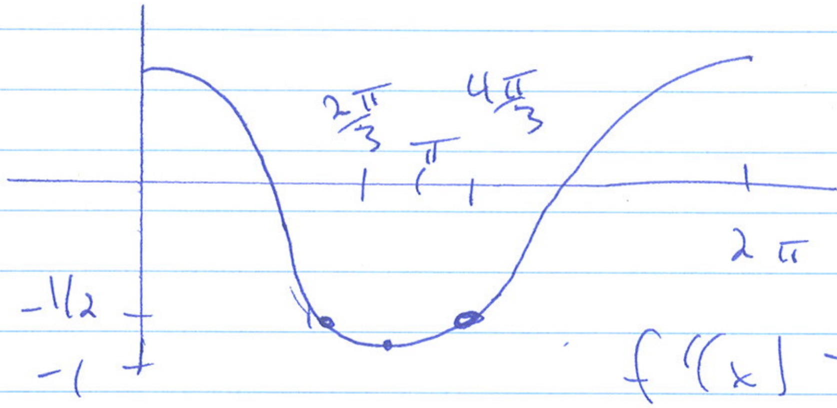
At $\frac{2\pi}{3}$ we have local max

At $\frac{4\pi}{3}$ " " " min.

(6)

On: $\left(0, \frac{2\pi}{3}\right)$: $f'(x) = 1 + 2\cos x$.

Clicker Q: → A: $f'(x) > 0$
B: $f'(x) < 0$.

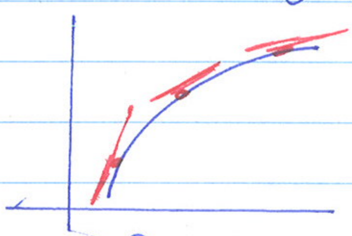


$\cos x > -1/2$.
$2\cos x > -1$
$1 + 2\cos x > 0$.

$f'(x) > 0$.

Concavity: Now two functions that are increasing can look very different.

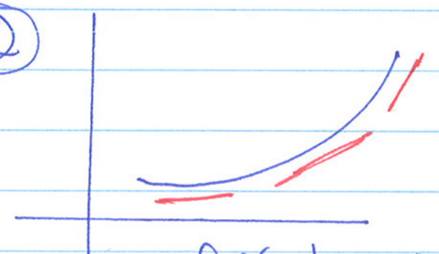
(1)



Concave Down.

$f'(x) > 0$
$f''(x) < 0$

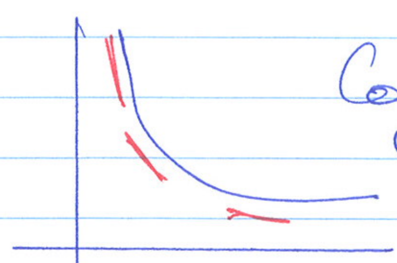
(2)



Concave Up.

$f'(x) > 0$
$f''(x) > 0$

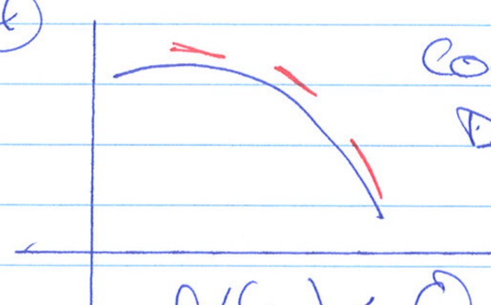
(3)



Concave Up

$f'(x) < 0$
$f''(x) > 0$

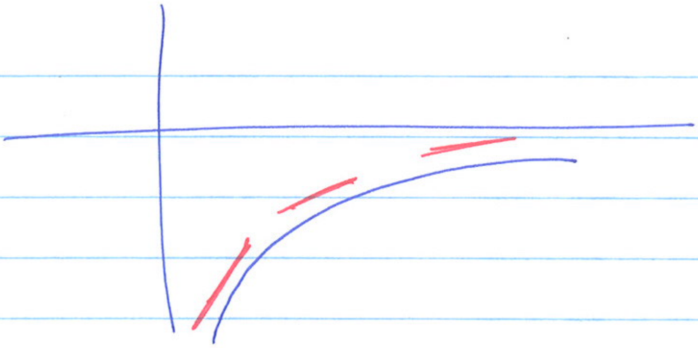
(4)



Concave Down.

$f'(x) < 0$.

$f''(x) < 0$.



$f'(x) > 0$.
$f''(x) < 0$.

$$\begin{cases} f''(x) > 0 \iff \text{Concave Up} \\ f''(x) < 0 \iff \text{Concave Down.} \end{cases}$$

Clicker Q: Which of the two are speeding up?

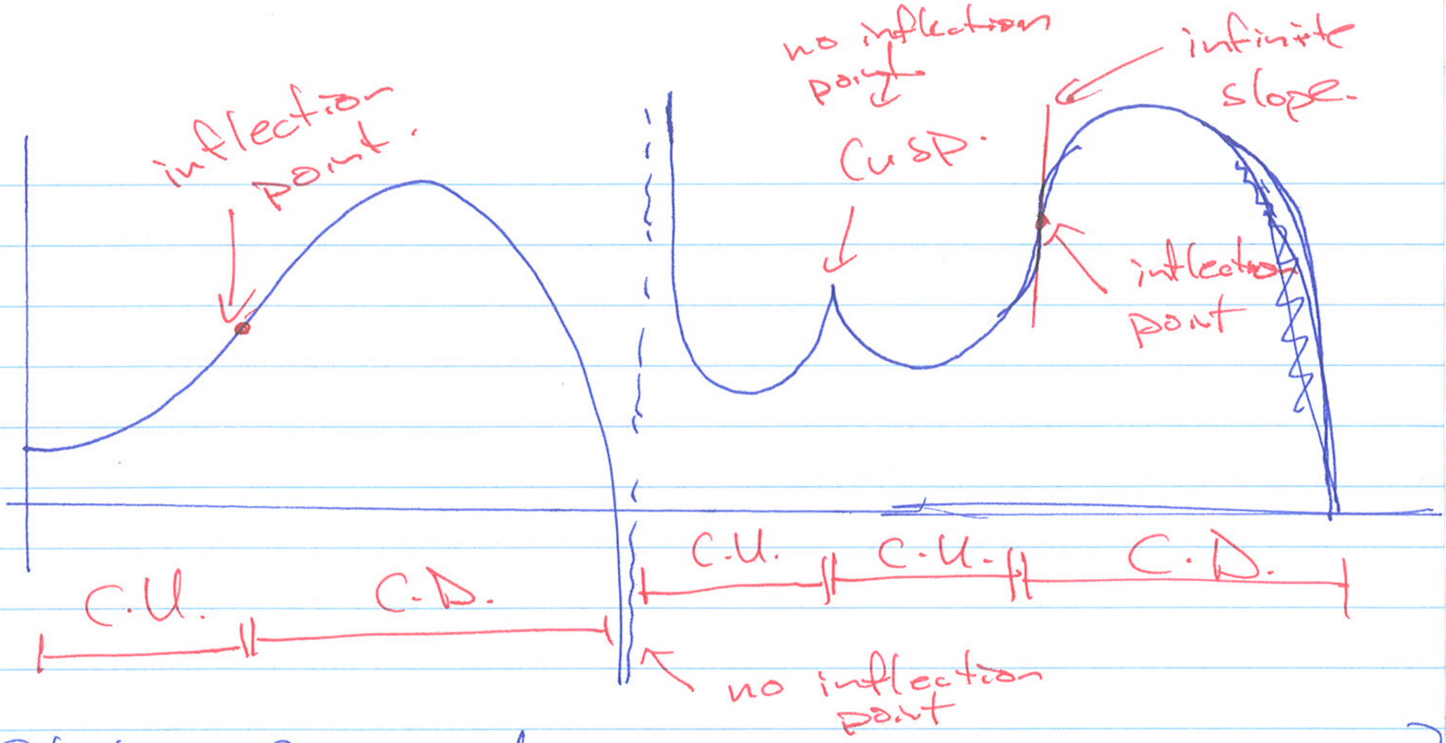
A: 1 and 2
B: 2 and 3
C: 2 and 4
D: 3 and 4.

2, 4 speeding up

1, 3 slowing down.

Definition: A point on the graph is called an inflection point if f(x) switches concavity at that point.

(8)



Clicker Q: How many inflection points?

A: 1
B: 2
C: 3
D: 4.

We can also use concavity to find out if a critical point is a local max/min ~~or neither~~.

Second Derivative Test:

If

$\cancel{f(x) = 12x^3 - 12x^2 - 24x}$

$f''(x) > 0$ at a critical point.

$\Rightarrow$ local min

$f''(x) < 0$ at a critical point.

$\Rightarrow$ local max.

(9).

Before we had.

$$f'(x) = 12x^3 - 12x^2 - 24x.$$

We found local min at $x = -1$.
using F.D.T.

OR/

$$f''(-1) = \dots = 12 \cdot 3 > 0.$$

$\Rightarrow$ local min.