1 March 5.

Midterm: March 12 in class
60 min.

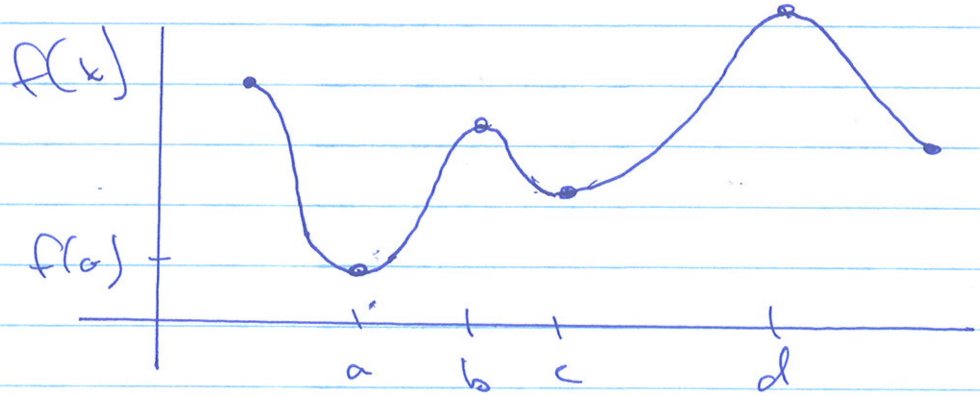
Office Hours: in LSK 300C.

Mon: 12 - 1:30
Tues: 2:30 - 4
Wed: 9:30 - 11.

Midterm Material:
§1.6 Inverses / Logarithms
§3.5 Implicit Diff
§3.6 Derivatives of Log.
§3.7 Rates of Change
§3.8 Exponential Growth/Decay
§3.9 Related Rates
§3.10 Linear Approx.
Course Notes: §1 §2 Taylor Polynomial with Remainder.

If you forget midterm 1 material, you're going to have a bad time.

§4.1 Maximum and Minimum Values



Points a, b, c, d are important in understanding the behaviour of $f(x)$.

$f(a)$ is the absolute minimum of f. the absolute min occurs at $x = a$.

Definition: Let c be in the domain of f. Then $f(c)$:

- is ~~the~~ an absolute minimum if $f(c) \leq f(x)$ for all x.
- is an absolute maximum if $f(c) \geq f(x)$ for all x.

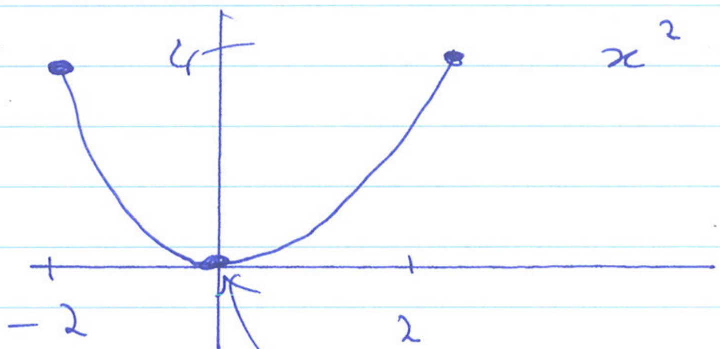
Global max/min is just another word for abs. max/min.

Def: $f(c)$ is a

- local max if $f(c) \geq f(x)$ for x near c.
- local min if $f(c) \leq f(x)$ for x near c.

Textbook: endpoints are not local max/min.

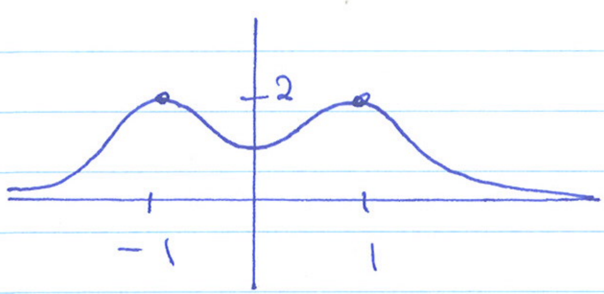
Ex:



Global max of 4. Occurs at endpoints.

local min and global min.

Clicker Q: Does this function have an abs. max?



→ A: Yes
B: No.
C: Don't know.

So $y = 2$ is the highest value reached by the function.

This max occurs at two points.

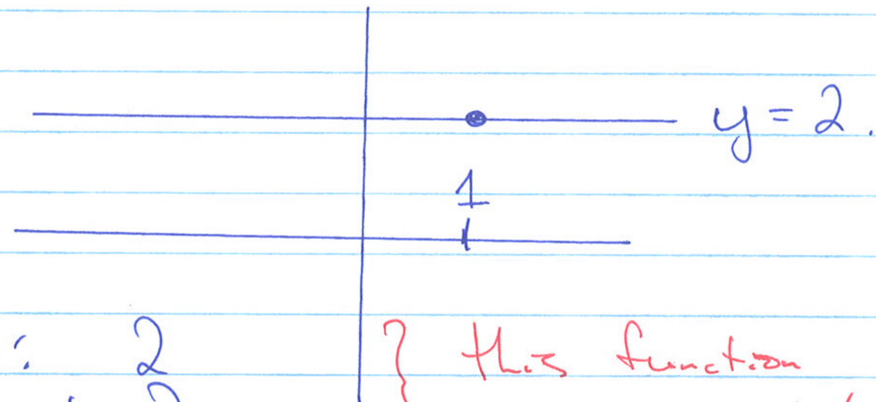
Clicker Q: Is there a global min?

A = Yes

B = No.

This function has no global min.

Example:



Abs. max: 2

Abs. min: 2.

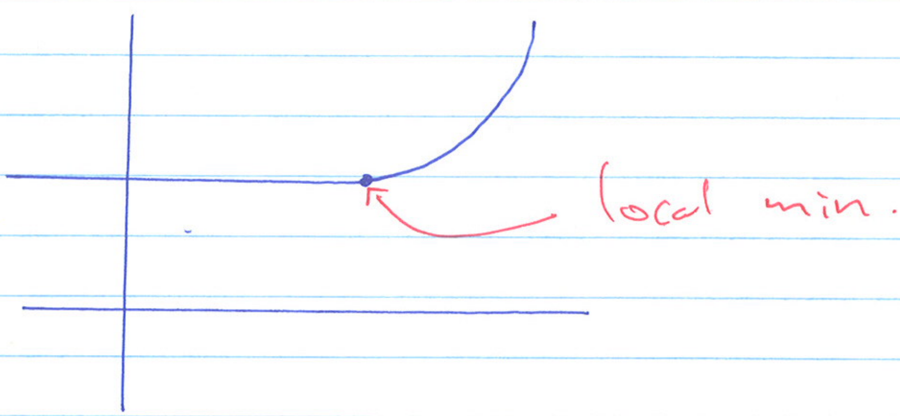
} this function has the same global max and min.

Is there a local min at $x = 1$?

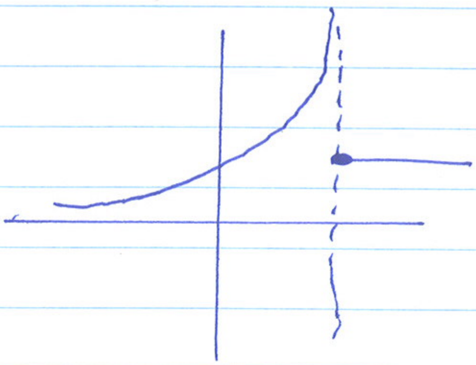
A: Yes

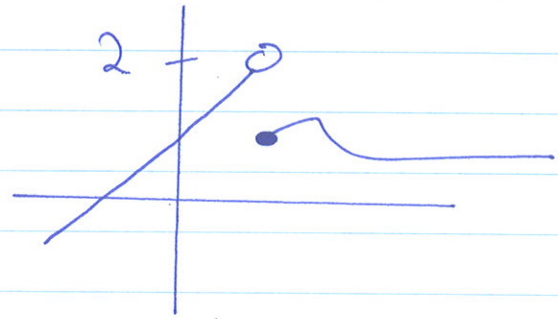
B: No.

Example:



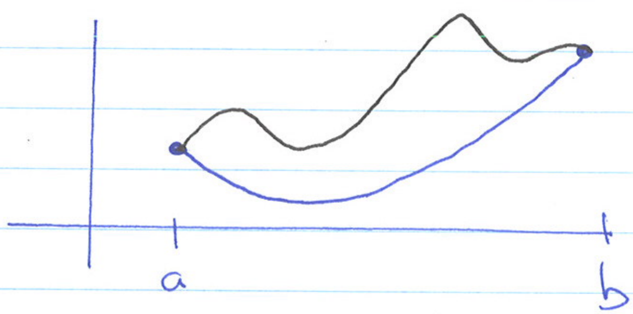
Does a function have to have a global max? No.





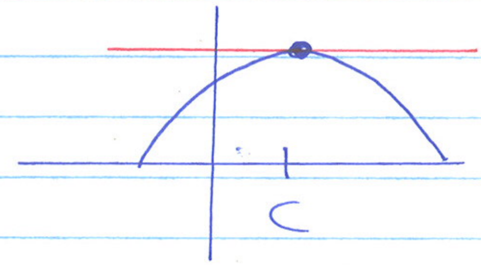
~~Ex~~ Extreme Value Theorem

A continuous function on a closed interval attains an absolute max and an absolute min.



The proof is way to hard.

Sometimes local/global max/min occur when $f'(c) = 0$.

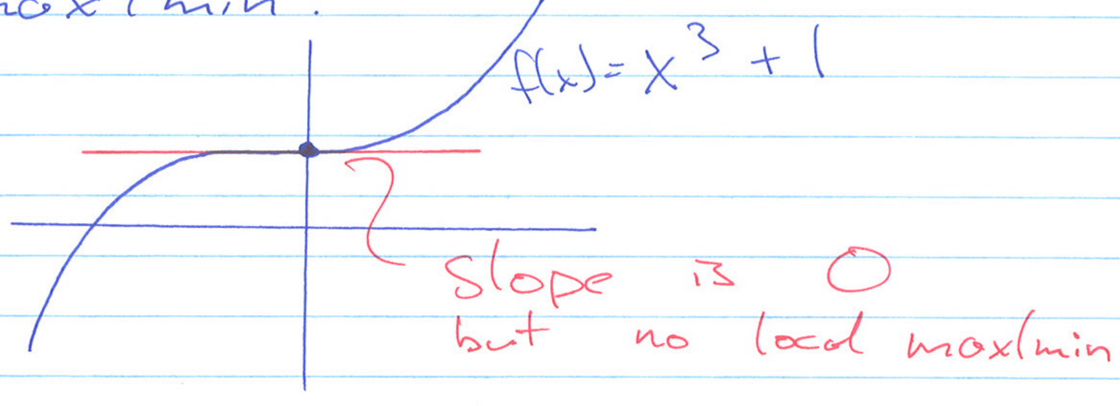


~~Fact~~ Fermat's Theorem:

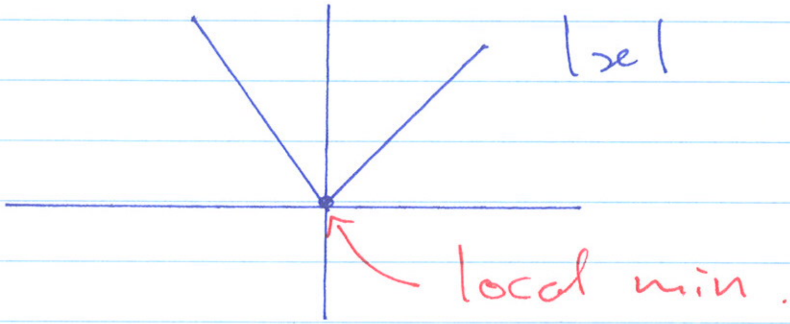
If f has a local max/min at c and if $f'(c)$ exists then $f'(c) = 0$.

So, if we are looking for max/min a good place to start is where $f'(x) = 0$.

Just because $f'(x) = 0$ it doesn't mean that we necessarily have a max/min.



We can also have points where $f'(x)$ does not exist.
These points may or may not be max/min.

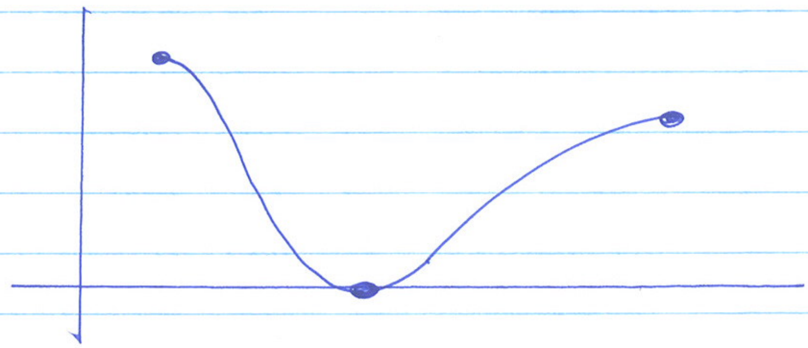


Points where $f'(x) = 0$ or where $f'(x)$ D.N.E. are important.
We call them critical points.

From Fermat's Theorem we get that: all local max/min are critical points.

Note: You can have critical points that are not local max/min.

Now a global max/min is either a local max/min or an endpoint.



If we want to find the abs. max/min we can just find all the critical points and endpoints and see which one is biggest.

In order to guarantee an abs. max and min we need a continuous function on a closed interval.

Closed Interval Method.
For a continuous function on a closed interval, to find abs. max and min, check all critical points and endpoints and see which is biggest/smallest.

Example: Find the abs. max and min of

$$f(x) = x^3 - 3x^2 + 1 \quad \text{on } -\tfrac{1}{2} \le x \le 4.$$

$$f'(x) = 3x^2 - 6x$$
$$= 3x(x-2)$$

$f'(x) = 0$ when $x = 0, 2$.
$f'(x)$ exists everywhere.

Endpoints: $x = -1/2, 4$.

$$f(-1/2) = -1/8 - 3/4 + 1 = \boxed{1/8}.$$

$$f(0) = \boxed{1}, \quad f(2) = 8 - 3\cdot 4 + 1 = \boxed{-3}.$$

$$f(4) = 4\cdot 16 - 3\cdot 16 + 1$$
$$= 16 + 1 = \boxed{17}.$$

Global max is: 17
occurs at $x = 4$.

Global min is: -3
occurs at $x = 2$.

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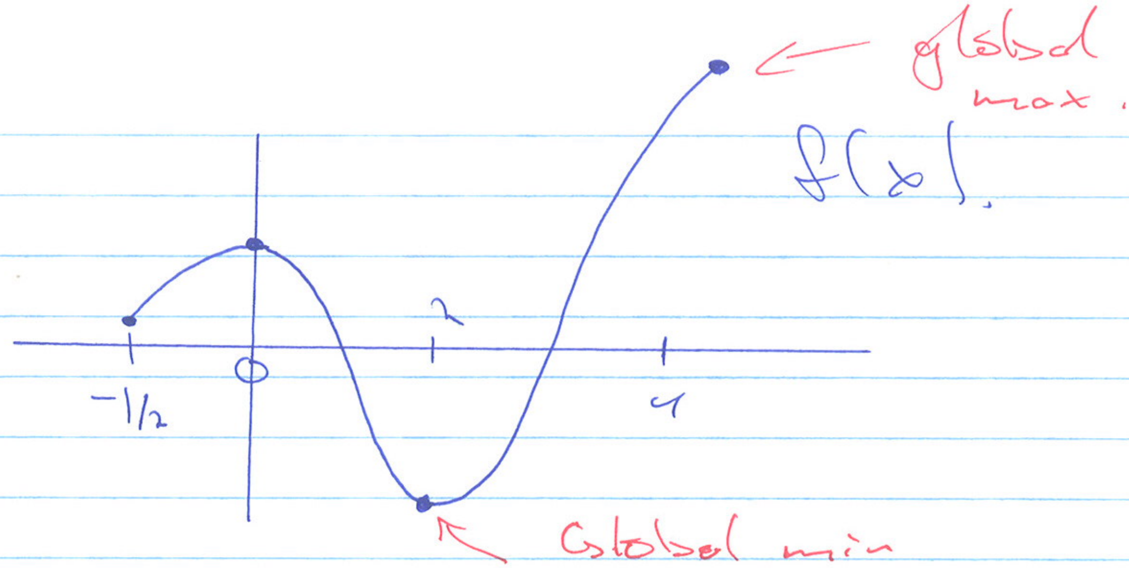

Example: Find abs. max/min of

$$f(x) = x^{3/5}(4-x) \text{ on } [-1,4].$$

Try this later.