Midterm 2:

1. c) $f(x) = \ln\left(\frac{1}{x\sqrt{3x+1}}\right)$.

$$f'(x) = \frac{1}{\frac{1}{x\sqrt{3x+1}}} \cdot \left(\left(x\sqrt{3x+1}\right)^{-1}\right)'$$

$$\left(x\sqrt{3x+1}\right)^{-1} \to -\left(x\sqrt{3x+1}\right)^{-2}\left(\sqrt{3x+1} + x\frac{1}{\sqrt{3x+1}}\cdot 3\right)$$

$$-\left(x\sqrt{3x+1}\right)^{-2}\left(\sqrt{3x+1} + \frac{3x}{\sqrt{3x+1}}\right)$$

$$f(x) = \cancel{\ln(1)}^{0} - \ln\left(x\cdot\sqrt{3x+1}\right)$$

$$= -\ln x - \ln\left(\sqrt{3x+1}\right)$$

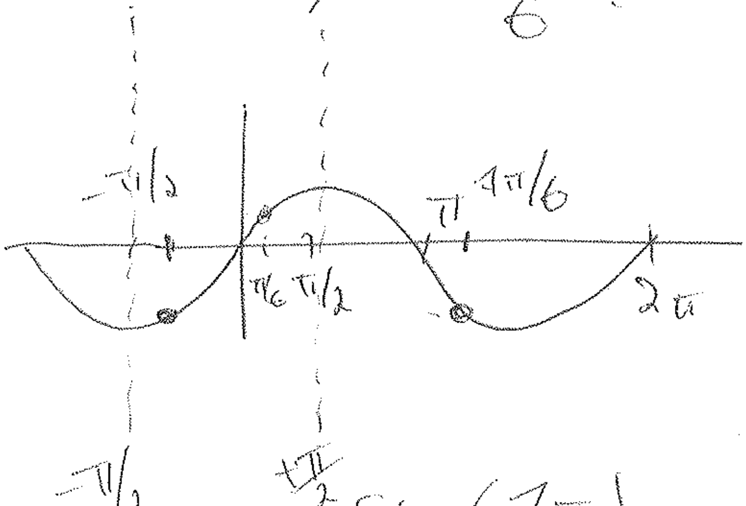
$$= -\ln x - \frac{1}{2}\ln(3x+1).$$

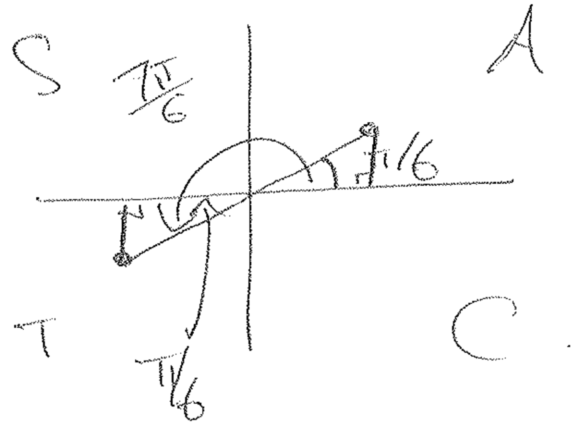
$$f'(x) = \frac{-1}{x} - \frac{1}{2}\frac{1}{3x+1}\cdot 3$$

$\arcsin(\sin x) = x \quad , \; x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

1. b) $\arcsin\left(\sin\left(\frac{7\pi}{6}\right)\right)$.

$\neq \frac{7\pi}{6}$.





$$\sin\left(\frac{7\pi}{6}\right) = -\sin(\pi/6)$$
$$= \sin(-\pi/6).$$

$$\arcsin\left(\sin\left(\frac{7\pi}{6}\right)\right)$$
$$= \arcsin(\sin(-\pi/6)) = -\pi/6$$

1. b) $f(x) = (\sin x)^{\ln x}$

$f'(x) = ?$

in class $\left(f(x) = x^x \right)$.

$$y = (\sin x)^{\ln x}$$

$$\ln y = \ln\left[(\sin x)^{\ln x} \right].$$

$$\ln y = \ln x \cdot \ln(\sin x).$$

take d/dx:

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(\sin x) + \ln x \frac{1}{\sin x}\cos x.$$

$$\frac{dy}{dx} = (\sin x)^{\ln x}\left[\frac{1}{x}\ln(\sin x) + \ln x \frac{1}{\sin x}\cos x \right].$$

2. a) $\sqrt[3]{25}$.

$L(x) = f(a) + f'(a)(x-a)$.

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\quad f(a) = \sqrt[3]{27}$

$= 3$.

$a = 27$.

$x = 25$.

$f'(x) = \frac{1}{3} x^{-2/3}$

$= \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$

$f'(27) = \frac{1}{3} \frac{1}{(\sqrt[3]{27})^2}$

$= \frac{1}{3} \cdot \frac{1}{3^2} = \frac{1}{27}$.

$L(x) = 3 + \frac{1}{27}(x - 27)$.

$\sqrt[3]{25} \simeq L(25) = 3 + \frac{1}{27}(25-27)$

$= 3 - \frac{2}{27}$.

2.b) $\frac{du}{dt} = u^2$.

A) $u(t) = 2t$

B) $u(t) = \frac{1}{3}t^3$

C) $u(t) = \frac{1}{1-t}$

D) $u(t) = e^{2t}$

E) None.

$u(t) = \frac{1}{3}t^3$.

$\frac{du}{dt} = t^2$.

$[u(t)]^2 = \left(\frac{1}{3}t^3\right)^2$.

C) $u(t) = \frac{1}{1-t} = (1-t)^{-1}$.

$\frac{du}{dt} = -(1-t)^{-2}(-1) = \frac{1}{(1-t)^2}$.

$= \left(\frac{1}{1-t}\right)^2$.

$= u^2$.

in class $\frac{dP}{dt} = aP$.

$P(t) = e^{at}$.

2.c) $e^{2y} - (y-1)\cdot x = 4$

tangent line at $(3,0)$.

$$(y-y_0) = m(x-x_0).$$

$$m = \frac{dy}{dx}.$$

take d/dx:

$$e^{2y} \cdot 2\frac{dy}{dx} - \frac{dy}{dx}x - (y-1) = 0.$$

$$\frac{dy}{dx}\left(2e^{2y} - x\right) = y-1$$

$$\frac{dy}{dx} = \frac{y-1}{2e^{2y} - x}.$$

$$\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{-1}{2-3} = \frac{-1}{-1} = 1.$$

$$(y-0) = 1(x-3)$$

2. d) $x \cdot \cos y + y \cdot \ln x = 0$.

$$\frac{dx}{dt} = 2/3$$

$$\frac{dy}{dt} = ? \quad \text{at} \quad (1, \pi/2).$$

take d/dt:

$$0 = \frac{dx}{dt}\cos y - x \sin y \frac{dy}{dt} + \frac{dy}{dt}\ln x + y\frac{1}{x}\frac{dx}{dt}$$

$$\frac{dy}{dt}(x\sin y - \ln x) = \frac{dx}{dt}\left(\cos y + \frac{y}{x}\right)$$

$$\frac{dy}{dt} = \frac{\frac{dx}{dt}\left(\cos y + \frac{y}{x}\right)}{(x\sin y - \ln x)}$$

$$= \frac{2/3\,(\pi/2)}{1 \cdot 1 - 0} = \pi/3$$

3. a) $\cot x = \dfrac{\cos x}{\sin x}$.

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x} = -\csc^2 x.$$

b) $\dfrac{d}{dx}\left(\operatorname{arccot} x\right) \stackrel{?}{=} \dfrac{-1}{1+x^2}$.

$y = \operatorname{arccot} x$

$\cot y = x$. take $\dfrac{d}{dx}$:

$-\csc^2 y \dfrac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{-1}{\csc^2 y} = \frac{-1}{1+\cot^2 y}$$

$$= \frac{-1}{1+x^2}.$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$1 + \cot^2 x = \csc^2 x.$

4)

0 hour : ? °C.

1 hour : 12°C

2 : 17°C.

Air : 25°C.

$\boxed{\frac{-13^2}{8}+25 = T(0).}$

$T(t) - Ts = (T(0) - Ts)e^{kt}$.

$Ts = 25$.

$T(1) = 12$

$T(2) = 17$.

$T(1) - Ts = (T(0) - Ts)e^{k\cdot 1}$

$-13 = 12 - 25 = (T(0) - 25)e^{k}$ (1)

$-8 = 17 - 25 = (T(0) - 25)e^{2k}$ (2)

Divide (2)/(1) : $\frac{8}{13} = \frac{e^{2k}}{e^{k}} = e^{k}$

$\Rightarrow k = \ln\left(\frac{8}{13}\right) < 0$.

(1):

$-13 = (T(0) - 25)e^{\ln(8/13)}$

$= (T(0) - 25)\, 8/13$.

$$-13 = (T(0) - 25)\frac{8}{13}.$$

$$-\frac{13^2}{8} = T(0) - 25.$$

$$25 - \frac{13^2}{8} = T(0)$$

$$\approx 3.875°C \text{ //}.$$

b) $t \to \infty$.

$$T(t) - T_S = (T(0) - T_S)e^{kt}.$$

$$k = \ln\left(\frac{8}{13}\right) < 0.$$

$$\lim_{t \to \infty} T(t) = T_S + \lim_{t \to \infty}(T(0) - T_S)e^{kt}.$$

$$= T_S + 0 = T_S.$$

Taylor: $T_2(x)$

5. a) $f(x) = x^{1/4}$ around $x = 1$.

$$T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2.$$

$a = 1$.

$f(1) = 1$.

$f'(x) = \frac{1}{4}x^{-3/4}$, $f'(1) = 1/4$.

$f''(x) = \frac{-3}{16}x^{-7/4}$, $f''(1) = -3/16$.

$$T_2(x) = 1 + \frac{1}{4}(x-1) - \frac{1}{2}\frac{3}{16}(x-1)^2.$$

5. b) $2^{1/4}$. $T_2(2) = \ldots$

$$|R_2(x)| \leq \frac{M}{3!}|x-a|^3.$$

$$|f'''(x)| \leq M \quad \text{on } [1,2]$$

$$\begin{cases} a = 1. \\ x = 2. \end{cases}$$

$$f'''(x) = \frac{3 \cdot 7}{64} x^{-11/4} = \frac{3 \cdot 7}{64} \frac{1}{x^{11/4}}$$

$$\leq \frac{3 \cdot 7}{64} \frac{1}{1^{11/4}} = \frac{21}{64}.$$

$$M = \frac{21}{64}.$$

$$|R_2(2)| \leq \frac{21}{64} \cdot \frac{1}{3!} (2-1)^3$$

$$= \frac{21}{64} \cdot \frac{1}{6}$$

$$R_2(x) = \frac{f'''(c)}{3!}(x-a)^3.$$

with c is in $[x, 0]$.

c). $|R_2(2)| \le \frac{\cancel{3}\cdot 7}{64}\frac{1}{\cancel{3}\cdot 2}$

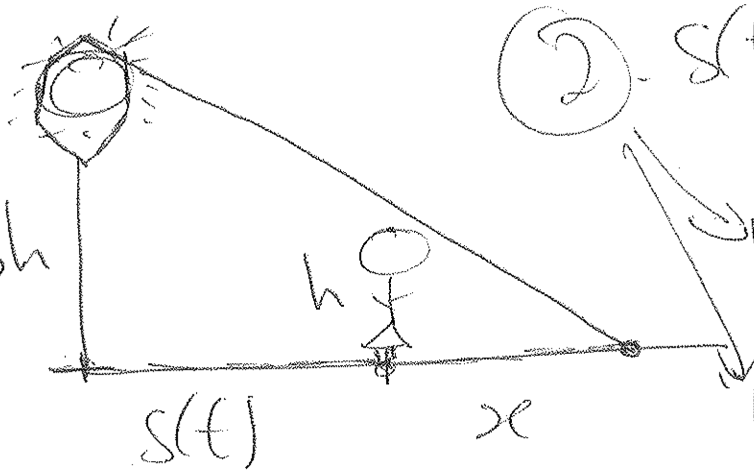
$$= \frac{7}{128} < \frac{10}{100} = 0.1.$$

$$0.1 = \frac{1}{10} = \frac{10}{100}$$

$$\frac{12.8}{128} //.$$

6.) a). Webwork.

1.



2. $s(t) = 4t - \frac{1}{2}t^2$.

Want: $\frac{dx}{dt}$?

"Move": $\frac{ds}{dt} = 1$.

at $t = 3$.

$$v(t) = \frac{ds}{dt} = 4 - t.$$

$$v(3) = 4 - 3 = 1.$$

3. Equation: $$\frac{s + x}{3h} = \frac{x}{h}.$$

$$s + x = 3x$$

$$s = 2x.$$

4. Chain Rule:

$$\frac{ds}{dt} = 2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{2} \,//.$$

4. b) Want $\frac{dx}{dt}$ after she has traveled 10m

Idea: $s(t) = 4t - \frac{1}{2}t^2.$

$$10 = 4t - \tfrac{1}{2}t^2.$$

$$0 = \tfrac{1}{2}t^2 - 4t + 10$$

$$= \tfrac{1}{2}(t^2 - 8t + 20).$$

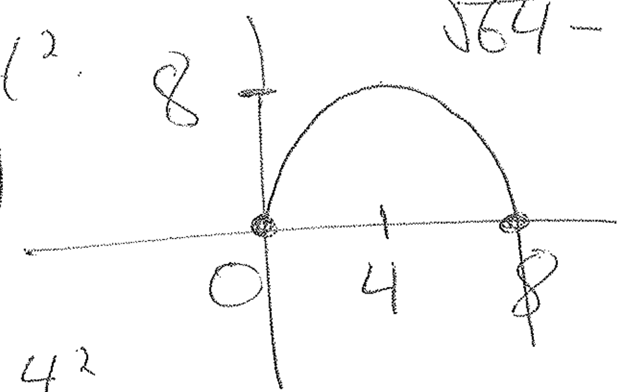
$$t = \frac{+8 \pm \sqrt{8^2 - 4\cdot 20}}{2}.$$

$$\cancel{\sqrt{4-20}} \ldots$$

$$\sqrt{64-80} \ldots$$

$$s(t) = 4t - \frac{1}{2}t^2.$$

$$= t\left(4 - \frac{1}{2}t\right)$$



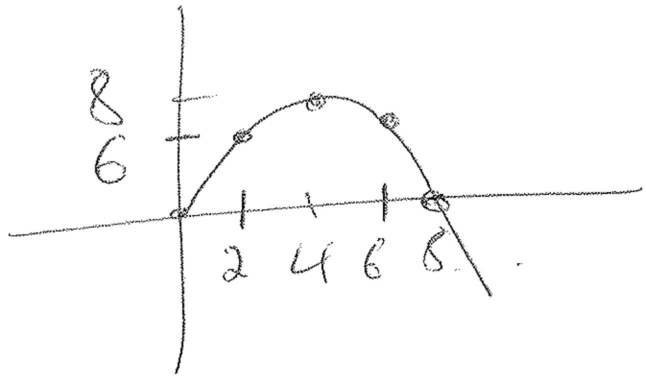
$$s(4) = 4\cdot 4 - \frac{1}{2}4^2$$

$$= 16 - 8 = 8.$$

10 m.

8m

2.



Find where $s(t) = 6$

$$6 = 4t - \frac{1}{2}t^2$$

$$\frac{1}{2}t^2 - 4t + 6 = 0$$

$$\tfrac{1}{2}(t^2 - 8t + 12) = 0$$

$$\tfrac{1}{2}(t-6)(t-2) = 0$$

$$t = 2, 6.$$

Take $\boxed{t = 6}$

Find $\frac{dx}{dt}$ when $t = 6$. $v(6) = 4 - 6 = -2$.

$$\frac{dx}{dt} = \frac{1}{2}\frac{ds}{dt} = \frac{1}{2}(-2) = -1.$$