3. If $f(x) = (1 + e^x)^2$, show that $f^{(k)}(0) = 2 + 2^k$ for any k. Write the Maclaurin polynomial of degree n for this function.

Solution: We have $f'(x) = 2e^x(1 + e^x)$ using the Chain Rule. We can rewrite $f'(x)$ as $2(e^x + e^{2x})$, and differentiating again gives $f''(x) = 2(e^x + 2e^{2x})$, $f'''(x) = 2(e^x + 4e^{2x})$, and in general $f^{(k)}(x) = 2(e^x + 2^{k-1}e^{2x})$ for $k \geq 1$. So, for $k \geq 1$, $f^{(k)}(0) = 2(1 + 2^{k-1}) = 2 + 2^k$. Since $f(0) = 4$, the Maclaurin polynomial of degree n is

$$T_n(x) = 4 + f^{(1)}(0)x + f^{(2)}(0)x^2/2! + \cdots + f^{(n)}(0)x^n/n! = 4 + 4x + 3x^2 + \cdots + (2 + 2^n)x^n/n!$$