

5. Find the Maclaurin polynomial of degree 3 for $f(x) = e^{\sin x}$.

Solution: Computing derivatives using various differentiation rules, we have $f'(x) = (\cos x)e^{\sin x}$, $f''(x) = -(\sin x)e^{\sin x} + (\cos^2 x)e^{\sin x} = (\cos^2 x - \sin x)e^{\sin x}$, and $f'''(x) = (-2(\cos x)(\sin x) - \cos x)e^{\sin x} + (\cos^3 x - (\sin x)(\cos x))e^{\sin x}$. Substituting $x = 0$ gives $f(0) = 1$, $f'(0) = 1$, $f''(0) = 1$, and $f'''(0) = 0$, so the third-degree Maclaurin polynomial is

$$T_3(x) = 1 + x + x^2/2$$