

1. Find the second degree Taylor polynomial about  $a = 10$  for  $f(x) = 1/x$  and use it to compute  $1/10.05$  to as many decimal places as is justified by this approximation.

*Solution:* Since  $f'(x) = -1/x^2$  and  $f''(x) = 2/x^3$ ,  $f(10) = 1/10$ ,  $f'(10) = -1/100$ , and  $f''(10) = 1/500$ , and so

$$T_2(x) = 1/10 - (x - 10)/100 + (x - 10)^2/1000$$

Also,  $f'''(x) = -6/x^4$ , and we have  $|f'''(t)| = 6/t^4 \leq 6/10^4$  for all  $t$  in the interval  $[10, 10.05]$ . Taking  $M = 6/10^4$ , we have

$$|R_2(10.05)| \leq (6/10^4)(10.05 - 10)^3/3! = 1/(10^4 \cdot 20^3) = (1/8) \cdot 10^{-7}$$

We have  $T_2(10.05) = 1/10 - .05/100 + (.05)^2/1000 = 0.0995025$ . Since the error bound above is less than  $(0.5) \cdot 10^{-7}$ , we can assert that  $1/10.05 \approx 0.0995025$  to 7 decimal places.