3. Show that the approximation

$$e = 1 + 1 + \frac{1}{2!} + \cdots + \frac{1}{7!}$$

gives the value of e to within an error of 8×10^{-5}.

Solution: The given approximation is the 7th-degree Maclaurin polynomial for e^x evaluated at $x = 1$. Since the 8th derivative of e^x is e^x, and the maximum (absolute) value of this 8th derivative on the interval $[0, 1]$ is e, the approximation has error at most $e \cdot (1 - 0)^8/8! = e/40320$. Since $e < 3$, the error is $< 3/40320$, which is $< 8 \cdot 10^{-5}$ as required.