

Math 190 Homework 8: Due Monday November 14

The assignment is due at the beginning of class on the due date. You are expected to provide full solutions, which are laid out in a linear coherent manner. Your work must be your own and must be self-contained. Your assignment must be stapled with your name and student number at the top of the first page.

Questions:

1. A spherical snow ball is melting such that its surface area is decreasing at a rate of $0.5\text{cm}^2/\text{min}$. How fast is the volume decreasing when the radius is 6cm ? the Volume and Surface Area of a sphere is given by

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad A = 4\pi r^2$$

respectively.

2. A boat is travelling down a river along the curve

$$y = \sqrt{x + 4}.$$

You notice that the y -coordinate of its position is decreasing at a rate of 10 knots when the boat is at the point $(-1/2, \sqrt{7/2})$. How fast is the x -coordinate of the boat changing at this point in time? Which is changing faster: the x -coordinate or the y -coordinate? Explain why you would expect this at the start of the problem, perhaps in reference to your picture.

3. Consider the function

$$f(x) = -\frac{1}{2}x + 5$$

- (a) Approximate the area under the curve on the interval $[0, 4]$ using Riemann Sums. Use left endpoints and two bars ($n = 2$).
 - (b) Now approximate the same area using four bars ($n = 4$), again with left endpoints.
 - (c) Compute the exact area, either by integrating or by drawing a picture and using area formulas. Which approximation is better? Are your approximations over or under estimates? Explain why you would expect this at the start of the problem, perhaps in reference to your picture.
4. Consider the function

$$g(x) = \begin{cases} 2(x - 1) + 6, & x \leq 1 \\ -x + 7, & x > 1 \end{cases}.$$

- (a) Sketch the graph of $g(x)$.
- (b) Compute

$$\int_{-1}^3 g(x) dx.$$

5. Compute the following definite integral

$$\int_0^\pi (\cos x + x^3 + 4) dx.$$