Math 190 Learning Objectives

Students should be able to do the following by the end of each respective unit. Additions/changes may be made as the term progresses.

Weeks 1, 2, 3: Review of Functions
We review the following in weeks 1, 2 and 3. Students will need to perform these tasks throughout the term as we study calculus.

1. Find the equation of a line (either in $y = mx + b$ or $y - y_1 = m(x - x_1)$ form) given two points or one point and the slope.
2. Plot a line given its equation (including x and y intercepts).
3. Find the roots of a quadratic equation either by factoring or quadratic formula.
4. Plot the graphs of x^2, x^3, $\sin x$, $\cos x$, $\tan x$, e^x, b^x, $\ln x$, $\log_b x$. Plot the graph of a piecewise function with simple parts.
5. Explain what a function is.
6. Find the domain of rational functions and functions involving square roots.
7. Recognize a given function as a composition of two (or more) simpler functions.
8. Use the unit circle and/or special triangles to compute sine, cosine and tangent of special angles as well as solve equations involving sine, cosine and tangent.
9. Simplify expressions involving exponents or logarithms using exponent or logarithm rules.
10. Solve equations involving exponents or logarithms by recognizing b^x and $\log_b x$ as inverses.

Week 4: Limits and Asymptotes

1. Explain using a picture what $\lim_{x \to a} f(x) = L$ means.
2. Explain using a picture what $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$ means.
3. Evaluate limits using the graph of a function.
4. Evaluate limits of rational, trigonometric, exponential and logarithmic functions.
5. Explain (either with a picture or with limits) the definition of vertical and horizontal asymptotes.
6. Given a function (rational or simple trig/exp/log) find the equations of any vertical and horizontal asymptotes.

Week 5: Definition of the Derivative

1. State the limit definition of the derivative.
2. Explain (using a picture) why the expression

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   should give the slope of the tangent line to $f(x)$ at given point x.
3. Use the limit definition of the derivative to find the derivative of simple functions: polynomials, rational functions, functions with square roots.
4. Given the graph of a function, sketch the graph of its derivative by considering various tangent lines of the given function.

Week 6: Derivative Rules

1. Use Power Rule to find the derivative of polynomials.
2. Compute the derivatives of sine, cosine and exponential functions.
3. Use Produce Rule and Quotient Rule to compute the derivatives of functions which are combinations of polynomials, trig functions, exponential functions.
4. Demonstrate the consistency between Power, Product and Quotient rules.
5. Use the derivative to find the slope and equation of tangent lines.
6. Find where a given function has horizontal tangent lines.

Week 7 and 8: Chain Rule and Related Rates

1. Compute the derivatives of compositions of functions using chain rule.
2. Combine chain rule with other differential rules (including chain rule itself) to compute the derivative of more complicated functions (involving: polynomials, exponential functions, logarithms, trig functions).
3. Find the equations of tangent lines of above mentioned functions.
4. Solve simple related rates problems using a systematic method of enumerated steps.

Week 9: Integrals and Riemann Sums

1. Approximate the area under a curve using a finite number of Riemann Sums. In simple cases argue why such an approximation may be an overestimate or an underestimate.

2. Explain using a picture why the expression

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

should give exactly the area under the curve $f(x)$. Indicate on your picture what n, x_i, $f(x_i)$ and Δx are.

Week 10: Definite and Indefinite Integrals

1. State the following part of the Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

explaining what $F(x)$ is.

2. Use integral rules to simplify/evaluate integrals.
3. Compute definite integrals of polynomials, trig functions and the exponential function.
4. Find the indefinite integral (general anti-derivative) of polynomials, trig functions and the exponential function.
5. Given a function, find a specific anti-derivative satisfying a certain condition.
6. Explain why a function has more than one anti-derivative.

Week 11: Method of Substitution

1. Perform the method of substitution to compute definite and indefinite integrals.
2. Identify possibilities for a substitution and demonstrate their usefulness/un-usefulness.
3. Explain, using an example, why substitution is the opposite of chain rule.

Week 12: Integration by Parts and Applications of Integrals

1. Compute definite/indefinite integrals using Integration by Parts.
2. Explain, using an example, why integration by parts is the opposite of product rule.
3. Solve simple physics problems, involving displacement, velocity and acceleration (or more generally, a quantity and it's rate of change), using anti-derivatives.

Course Wide:

1. Sketch the graph of a function satisfying certain properties. These properties could include information about the limits, asymptotes, derivative, integral of the function.