1

Substitution Method.

For more complicated integral's we need more complicated methods.

Example: $\int (\underbrace{2x+1}_{u})^2 \, dx$.

Rather than guessing, let's make a substitution.

Let us call $u = 2x+1$.

$$\int (2x+1)^2 \, dx = \int u^2 \, dx$$

change all x's to u's.

this integral is still in terms of x.

get rid of dx in favour of du.

$u = 2x+1$ → $du = 2dx$.

$$\frac{du}{dx} = 2.$$

$$dx = \frac{1}{2} du.$$

found dx in terms of du.

②

So, $\int (2x+1)^2 \, dx$

$$= \int u^2 \frac{1}{2} \, du.$$

$$= \frac{1}{2} \int u^2 \, du.$$

$$= \frac{1}{2} \frac{1}{3} u^3 + C.$$

$$= \frac{1}{6} u^3 + C.$$

change back to original variable.

$$= \frac{1}{6} (2x+1)^3 + C.$$

check: $\left[\frac{1}{6} (2x+1)^3 + C \right]'$

$$= \frac{3}{6} (2x+1)^2 \cdot 2 + 0.$$

$$= (2x+1)^2.$$

Substitution is like anti-chain rule.

3

u.

Example: $\int e^{4x}\,dx$.

Let $u = 4x$?

~~$u = 2e^{x}$~~ ?

$u = 4x$

$\frac{du}{dx} = 4$. $\Rightarrow$ $du = 4\,dx$

$\frac{1}{4}du = dx$.

$$\int e^{4x}\,dx = \int e^{u}\frac{1}{4}\,du.$$

$$= \frac{1}{4}\int e^{u}\,du.$$

$$= \frac{1}{4}e^{u} + C.$$

$$= \frac{1}{4}e^{4x} + C.$$

Example:

$$\int x \sin(x^2)\, dx.$$

What substitution to use?

$\cancel{u = x\sin(x^2)}$. A

$u = x^2$ B

$\cancel{u = \sin(x^2)}$ C

$\cancel{u = x}$ D.

$$u = \sin(x^2).$$

$$\frac{du}{dx} = \cos(x^2)\cdot 2x.$$

$$\frac{du}{2x\cos(x^2)} = dx.$$

$$u = x^2 \quad \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx.$$

$$\int \sin(\underbrace{x^2}_{u.})\, \underbrace{x\,dx}_{\frac{1}{2}du.}.$$

$$= \int \sin(u)\,\frac{1}{2}\,du.$$