- Quiz 5 - Friday

2Q's $\left\{ \begin{array}{l} \text{- integrals} \\ \text{- substitution.} \end{array} \right\}$ 15 min.

- A harder substitution problem will be posted as a note.
- HW9 Due Monday.
- Past Exam and Exam Practice to be posted.

Example: One more substitution.

$$\int_1^2 \frac{\ln x}{x}\,dx = \int_1^2 \underbrace{\ln x}_{u} \overbrace{\frac{1}{x}\,dx}^{du}$$

derivative of $\ln x$.

Let $u = \ln x$.

$$\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}\,dx.$$

$$\int_{x=1}^{x=2} \ln x \frac{1}{x}\,dx = \int_{u=0}^{u=\ln 2} u\,du.$$

when $x = 1$, $u = \ln(1) = 0$.
when $x = 2$, $u = \ln(2)$.

$$= \frac{1}{2} u^2 \Big|_{u=0}^{u=\ln 2}.$$

$$= \frac{1}{2}(\ln 2)^2 - \frac{1}{2} 0^2.$$

$$= \frac{(\ln 2)^2}{2}.$$

Example: $\int x \cdot e^x \, dx.$

Integration By Parts (IBP)

$$\int u\,dv = uv - \int v\,du.$$

$$\int \overset{u}{x} \underbrace{e^x \, dx}_{dv}.$$

let $u = x$ $\qquad$ $dv = e^x dx.$

$\frac{du}{dx} = 1$ $\qquad$ $\int 1\,dv = \int e^x dx.$

$du = dx.$ $\qquad$ $v = e^x.$

$$\int x e^x \, dx = \int u \, dv$$

$$= uv - \int v \, du.$$

$$= x e^x - \int e^x \, dx.$$

$$= x e^x - e^x + C.$$

Check:

$$\left(x e^x - e^x + C \right)'$$

$$= \cancel{1 \cdot e^x} + x e^x \cancel{- e^x} \cancel{+ 0}$$

$$= x e^x$$

Integration by parts is like anti-product rule.

Notice: • we differentiated u
• we integrated dv.

Example: $\int x \cos x \, dx$.

Let $u = x$ $\quad$ $dv = \cos x \, dx$.

$du = dx$ $\quad$ $v = \int \cos x \, dx$

$v = \sin x$.

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use: $\int u\,dv = uv - \int v\,du$.

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$
$$= x\sin x + \cos x + C.$$

Example: $\int x\ln x\,dx$.

Clicker Q:

33% A) $u = x$ $\quad dv = \ln x\,dx$

67% B) $u = \ln x$ $\quad dv = x\,dx$.

Try A) $u = x$ $\quad dv = \ln x\,dx$

$du = dx$

diff. this is easy ...but... int is hard. $\quad v = \int \ln x\,dx$.

Try B) $u = \ln x$ $\quad dv = x\,dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = \int x\,dx$.

$du = \frac{1}{x}\,dx$. $\quad = \frac{1}{2}x^2$.

Use $\int u\,dv = uv - \int v\,du$.

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\frac{1}{x}\,dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx.$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\cdot\frac{1}{2}x^2 + C.$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C.$$

Example: $\displaystyle\int_0^{\pi/2} x^2\sin x\,dx$.

Definite integrals work in the same way:

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$$

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let $u = x^2 \qquad dv = \sin x\,dx$

$du = 2x\,dx \qquad v = -\cos x$.

$$\int_0^{\pi/2} x^2 \sin x\,dx = -x^2\cos x\Big|_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,2x\,dx.$$

$$= \overbrace{-x^2\cos x}^{\text{"}0\text{"}}\Big|_0^{\pi/2} + 2\int_0^{\pi/2} x\cos x\,dx.$$

$$= -\left(\frac{\pi}{2}\right)^2 \overbrace{\cos(\pi/2)}^{\text{"}0\text{"}} - \big(\underbrace{-0^2\cos(\pi/2)}_{\text{"}0\text{"}}\big)$$

$$+ 2\int_0^{\pi/2} x\cos x\,dx.$$

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \sin x$

$$= 2\left(x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x\,dx\right).$$

$$= 2\left(\frac{\pi}{2}\overbrace{\sin(\pi/2)}^{\text{"}1\text{"}} - \overbrace{0\cdot\sin(0)}^{\text{"}0\text{"}} + \cos x\Big|_0^{\pi/2}\right).$$

$$= \pi + 2\cos x\Big|_0^{\pi/2}.$$

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$$= \pi + \overbrace{2\cos(\pi/2)}^{0} - \underbrace{2\cos(0)}_{1}.$$

$$= \pi - 2.$$