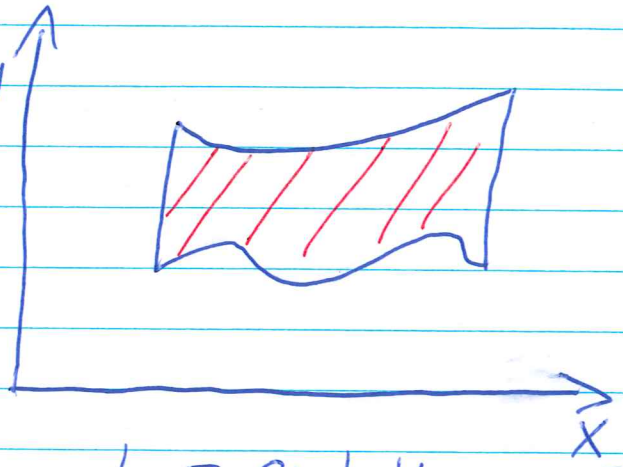
Nov 2nd

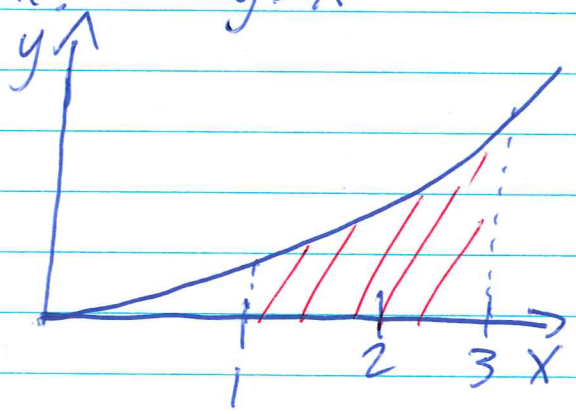
- Differential calculus
- Integral Calculus

Q:



How do I find its area?

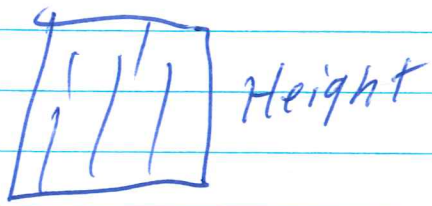
Working example: $y = x^2$



Q: Area between 1 and 3

→ use Rectangles!

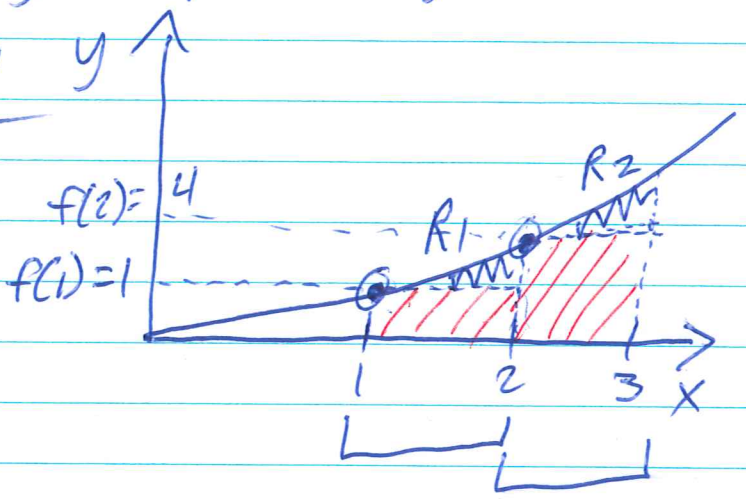
A = Height · base



Working example: $y = x^2$

Attempt 1

Left endpoint



base = 1

$R_1 = f(1) \cdot 1 = 1$

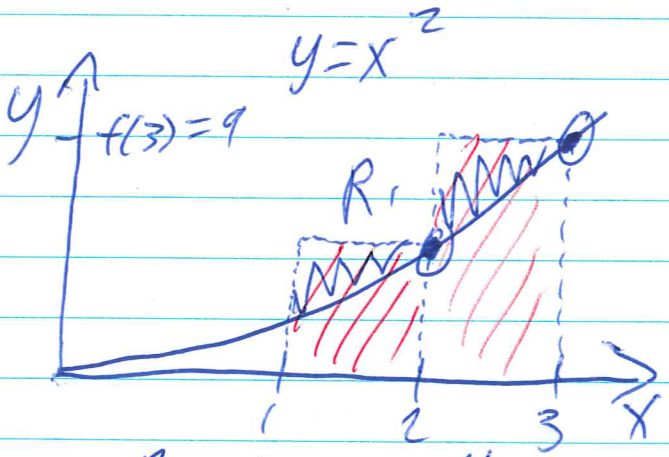
$R_2 = f(2) \cdot 1 = 4$

Total: $R_1 + R_2 = 5$

Did we miss anything?
Yes!

Attempt 2

Right endpoints



$R_1 = f(2) \cdot 1 = 4$

$R_2 = f(3) \cdot 1 = 9$

Total: $R_1 + R_2 = 13$

-> Actual is inbetween 5 and 13!

Problem! Not good enough approximation!

→ Fix: use more rectangles!

Notation: Σ ← capital sigma / summation

$$\sum_{i=1}^{2} i^2 = 1^2 + 2^2$$

end — 2 — rule

$i = 1$ — start

Index / dummy variable

5 — stop

$$\text{ex: } \sum_{i=2}^{5} \tfrac{1}{2}(1+i) = \tfrac{1}{2}(1+2) + \tfrac{1}{2}(1+3) + \tfrac{1}{2}(1+4) + \tfrac{1}{2}(1+5)$$

start

$$\text{ex 2: } \sum_{i=0}^{3} (1+i)^2 = (1+0)^2 + (1+1)^2 + (1+2)^2 + (1+3)^2$$

Left: $A = 5$
Right: $A = 13$

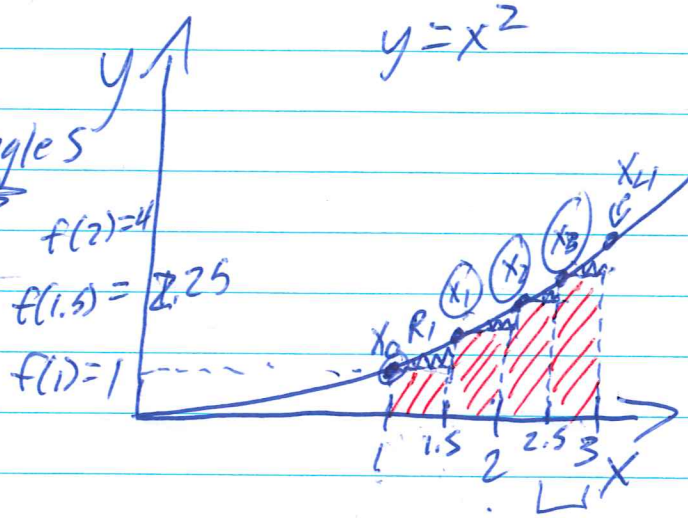
(4)

Spoiler: Actual Area: $A = 8.667$

$y = x^2$

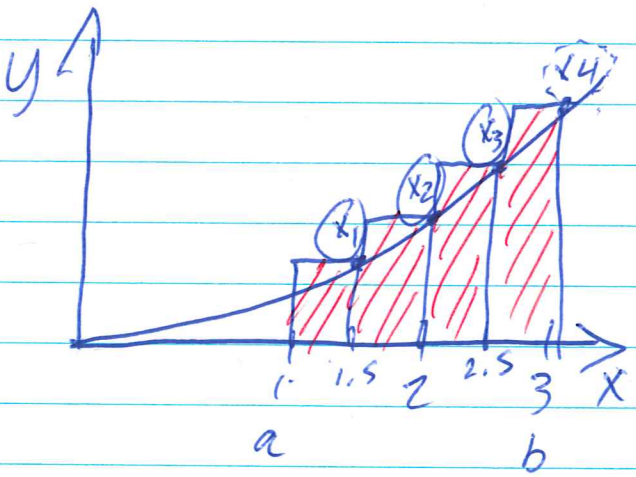
use 4 rectangles

Left endpoints



Base: .5
$\Delta x = .5$

$R_1 = 1 \cdot .5 = .5$
$R_2 = 2.25 \cdot .5 = 1.125$
$R_3 = 4 \cdot .5 = 2$
$R_4 = (2.5)^2 \cdot .5 = 3.125$
Total: $R_1 + R_2 + R_3 + R_4 = 6.75$



$\Delta x = .5$
$= \frac{3-1}{4} = \frac{2}{4} = .5$

Summarize:

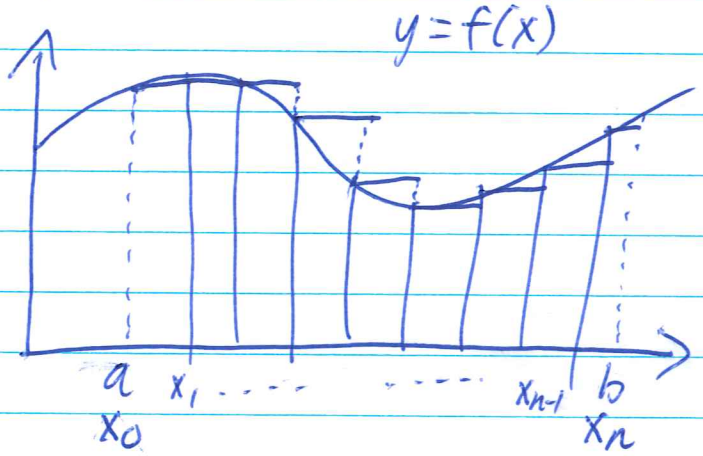
Left: $R_1 = f(x_0)\cdot\Delta x$
$R_2 = f(x_1)\cdot\Delta x$
$R_3 = f(x_2)\cdot\Delta x$
$R_4 = f(x_3)\cdot\Delta x$

$\Rightarrow \sum_{i=0}^{3} f(x_i)\cdot\Delta x$.

Right: $R_1 = f(x_1)\cdot\Delta x$
$\vdots$
$R_4 = f(x_4)\cdot\Delta x$

$\Rightarrow \sum_{i=1}^{4} f(x_i)\Delta x$

In general: for Left



$\Delta x = \frac{b-a}{n}$

Left:

$$\text{Area} = \sum_{i=0}^{n-1} f(x_i)\cdot\Delta x .$$

Right:

$$\text{Area} = \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

Riemann Sums