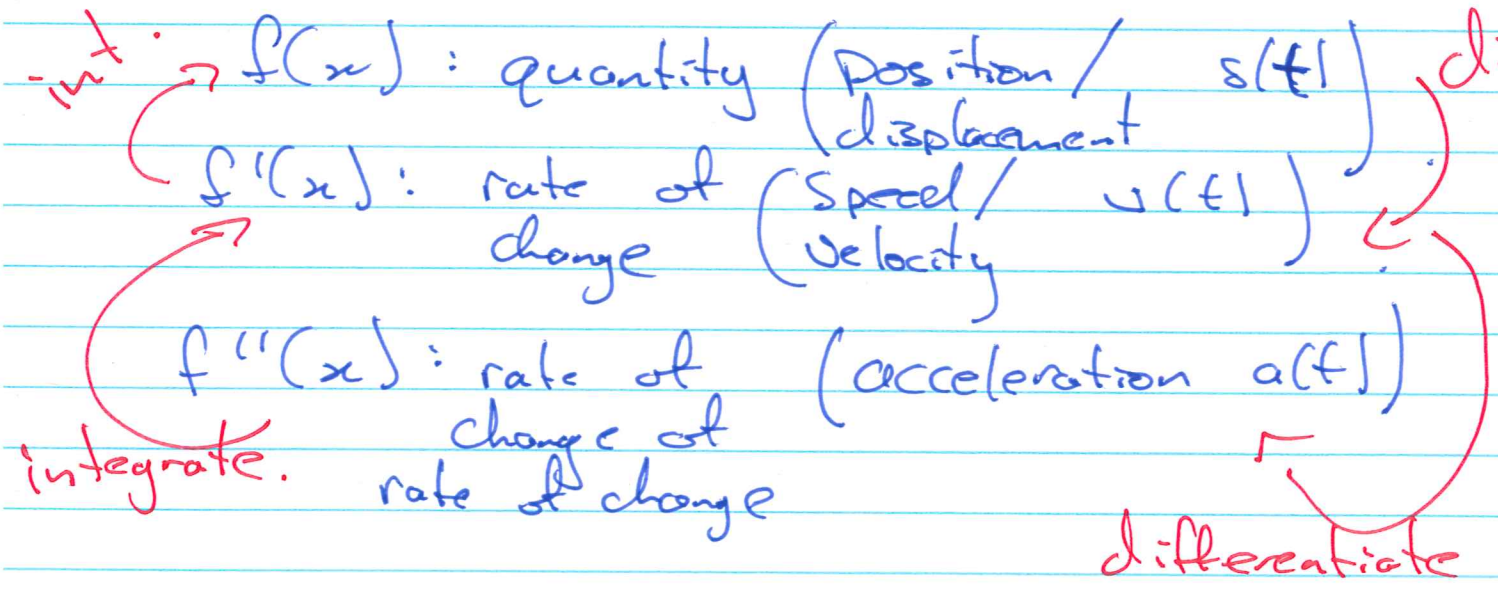
- Office Hours Today (Wednesday) 1-2:30pm at my office: AA 137.
- Quiz 5 Friday. — basic integrals — substitution.
- HW 10 - last HW! - due Monday
- Grades Updated.
- Course Eval Survey - do them!

Some Applications of Derivatives and Integrals.



Example: A runners distance from the starting block is given by

$$s(t) = t^2.$$ (seconds, metres).

(at least initially).

After how long does the runner hit a speed of 3 m/s?

We want velocity: $s(t) = t^2$

$v(t) = 2t.$

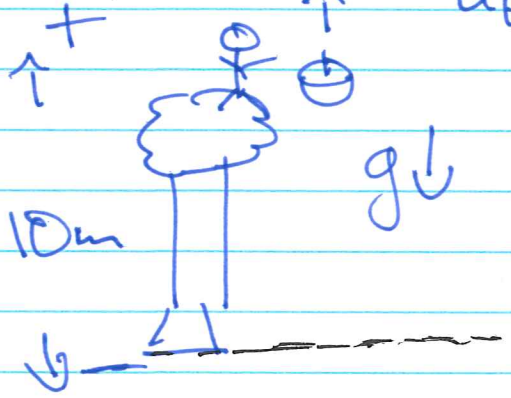
When does $3 = v(t) = 2t$.?

$$3 = 2t$$
$$t = 3/2 \text{ seconds}$$

Using these ideas we can solve all the high school physics problems (without memorizing anything).

(3)

Example: The acorn is thrown upwards with velocity 2 m/s.



1) When is the acorn (momentarily) at rest.
ie $v(t) = 0$.

2) When does the acorn hit the ground.

Assume acceleration due to gravity is constant $-9.8\ m/s^2$.

$$a(t) = -9.8$$

1) We want velocity. Integrate:

$$v(t) = \int a(t)\,dt = \int -9.8\,dt.$$

$$= -9.8t + C.$$

So,

$$v(t) = -9.8t + C.$$

how to find C.
use initial velocity.

(4)

We have $v(0) = 2$.

$$2 = v(0) = -9.8(0) + C.$$

$$\Rightarrow \quad C = 2.$$

So, $v(t) = -9.8t + 2$.

Now, when does $v(t) = 0$?

$$0 = -9.8t + 2.$$

$$9.8t = 2$$

$$t = \frac{2}{9.8} \text{ sec.}$$

$$\simeq 0.2 \text{ s.}$$

2). How long until the acorn hits the ground?

How to find $s(t)$? Integrate.

distance from the ground $\rightarrow s(t) = \int v(t)\,dt$.

(5)

$$s(t) = \int (-9.8t + 2)\,dt.$$

$$= \frac{-9.8}{2}t^2 + 2t + C.$$

find C. ↑ use initial distance.

have: $10 = s(0)$

distance from the ground at any time.

$$= \frac{-9.8}{2}(0)^2 + 2(0) + C.$$

$$C = 10.$$

$$s(t) = \frac{-9.8}{2}t^2 + 2t + 10.$$

When does $s(t) = 0$?

$$0 = \frac{-9.8}{2}t^2 + 2t + 10.$$

$$t = \frac{-2 \pm \sqrt{2^2 - 4\left(\frac{-9.8}{2}\right)(10)}}{2 \cdot \left(\frac{-9.8}{2}\right)}.$$

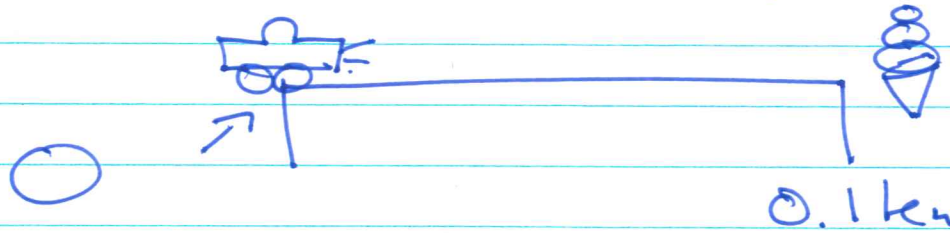
$t =$ ....

this gives negative time!

$$t = \frac{-2 \pm \sqrt{200}}{-9.8} \qquad \sqrt{200} = 10.$$

$$t = \frac{-2 - \sqrt{200}}{-9.8} = \frac{2 + \sqrt{200}}{9.8}.$$

Example: You are driving at 100km/h. You see someone selling ice cream 100m ahead and slam on the breaks. What constant acceleration is needed to stop in time?



$a(t) = -A$ ← we are slowing down.

$v(t) = -At + C$ ← initial velocity.

(4)

$$v(0) = 100. = -A \cdot 0 + C.$$

$$C = 100.$$

$$v(t) = -At + 100.$$

$$s(t) = \ldots$$

$$s(t) = \int v(t)\,dt.$$

$$s(t) = \frac{-A}{2}t^2 + 100t + C$$

Initial Position? ~~s(t)~~ $s(0) = 0.$

$$0 = \frac{-A}{2}0^2 + 100(0) + C$$

$$\Rightarrow C = 0.$$

want A.

$$s(t) = \frac{-A}{2}t^2 + 100t$$

need t when we stop
need $s(t)$ when we stop.

When do we stop? when $v(t) = 0$.

or.

$$0 = -At + 100$$

$$t = \frac{100}{A}.$$

(8)

Let us call this special stopping time t^*.

$$t^* = \frac{100}{A}.$$

We would like the car to stop after 0.1 km, that is.

Now we can solve for A.

$$s(t^*) = 0.1 = -\frac{1}{2} A (t^*)^2 + 100 \cdot t^*$$

$$0.1 = s\left(\frac{100}{A}\right) = -\frac{1}{2} A \left(\frac{100}{A}\right)^2 + 100 \cdot \frac{100}{A}.$$

$$\frac{1}{10} = \frac{-1}{2} \frac{A \cdot 100 \cdot 100}{A^2} + 100 \cdot \frac{100}{A}.$$

$$\frac{1}{10} = \frac{-50 \cdot 100}{A} + \frac{100 \cdot 100}{A}.$$

$$\frac{A}{10} = -50 \cdot 100 + 100 \cdot 100$$

$$= (-50 + 100)\, 100 = 50 \cdot 100$$

$$A = 500 \cdot 100$$

$$= 50\,000 \text{ km/h}^2 \text{ or } \sim 3.858 \text{ m/s}^2.$$

Need a constant acceleration of $-50\,000$ km/h^2 to get ice cream.