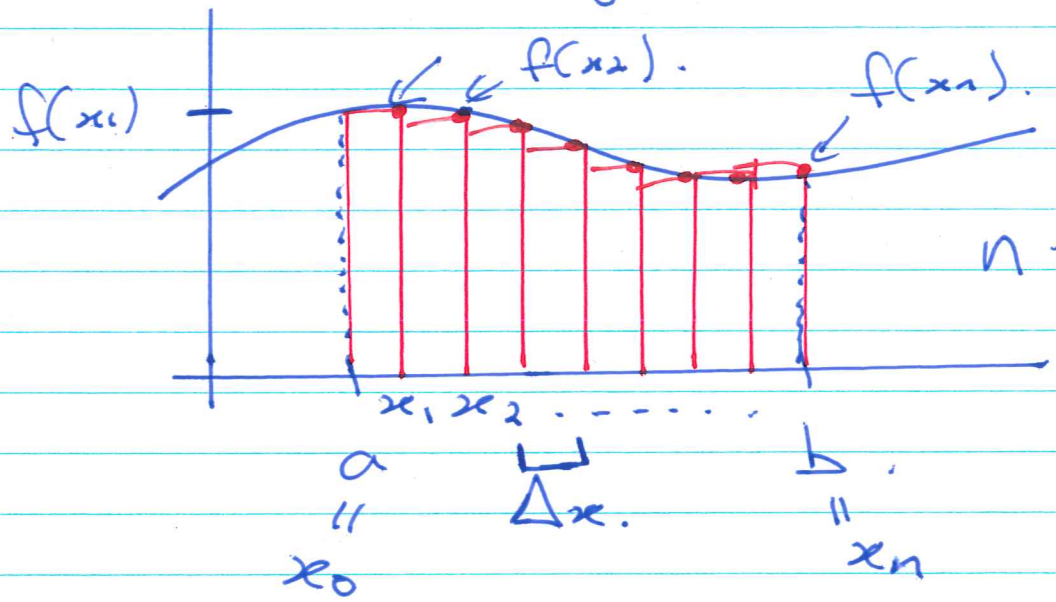
- Midterm solutions posted
  - back Monday?
- Quiz 4 Nov. 14
- HW8 posted monday
  - due Nov. 14.

Last Class: We learned to approximate the area under curves using Riemann Sums.



n - # of bars.

$$\Delta x = \frac{b-a}{n}.$$

The area is approximately,

$$f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_{n-1})\Delta x + f(x_n)\Delta x.$$

$$= \sum_{i=1}^{n} f(x_i)\Delta x$$

(using right endpoints)

To make the approximation better? Use more bars!

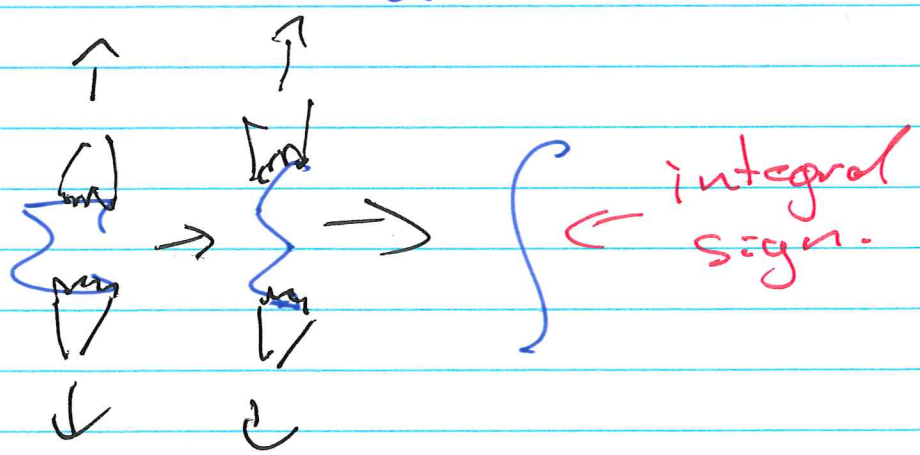
To get the area exactly we take the limit as $n \to \infty$.

The exact area:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx.$$

← area under $f(x)$ between a and b.

↑ number of bars goes to infinity.



← integral sign.

(3)

$$\int_a^b f(x)\,dx$$

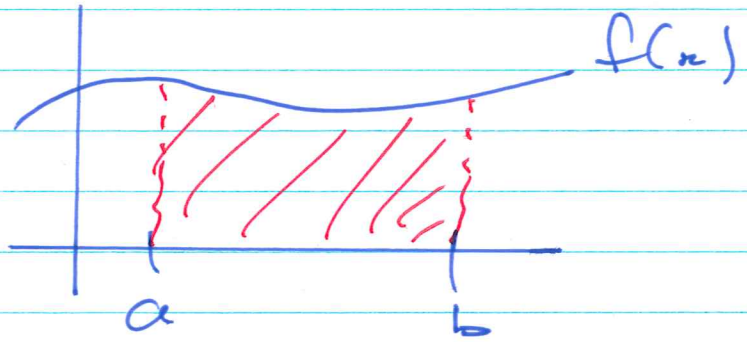
limits of integration

notation to tell us we are integrating in x.

"integrand" → the function we are integrating.

$\Delta x \to dx$

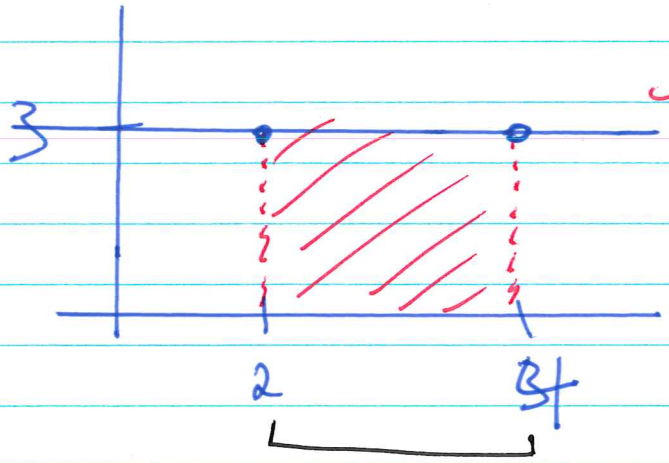
$\sum \to \int$



Clicker Q: What is $\int_2^4 3\,dx$?

A) 3 B) 10

→ C) 6 D) 12.



$4 - 2 = 2$.

area is $3 \cdot (4-2)$

$= 3 \cdot 2 = 6$.

In general. (k is a constant.)

$$\int_a^b k\,dx = k(b-a).$$

Clicker Q: Suppose we knew that

$$\int_0^3 x^2\,dx = 9.$$

What is $\int_0^3 2x^2\,dx$?

A) 9 → B) 18

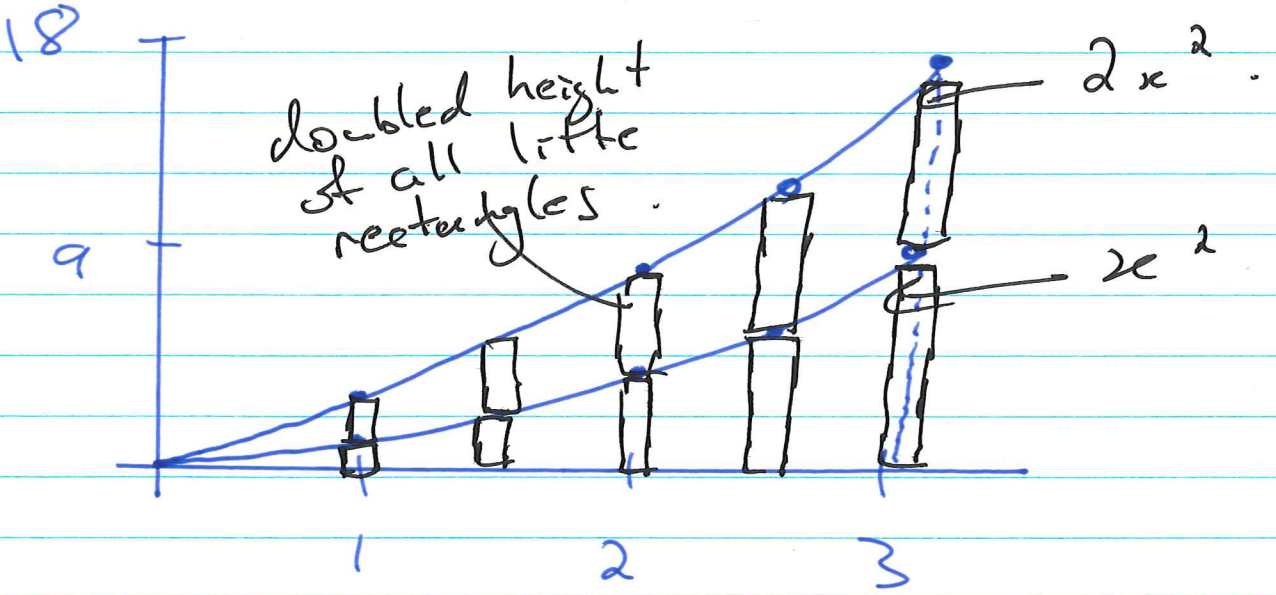
C) 27 D) 48.

(5)

Suggestion: Pull out constant.

$$\int_0^3 2x^2\,dx = 2\int_0^3 x^2\,dx.$$

$$= 2 \cdot 9 = 18.$$



In general,

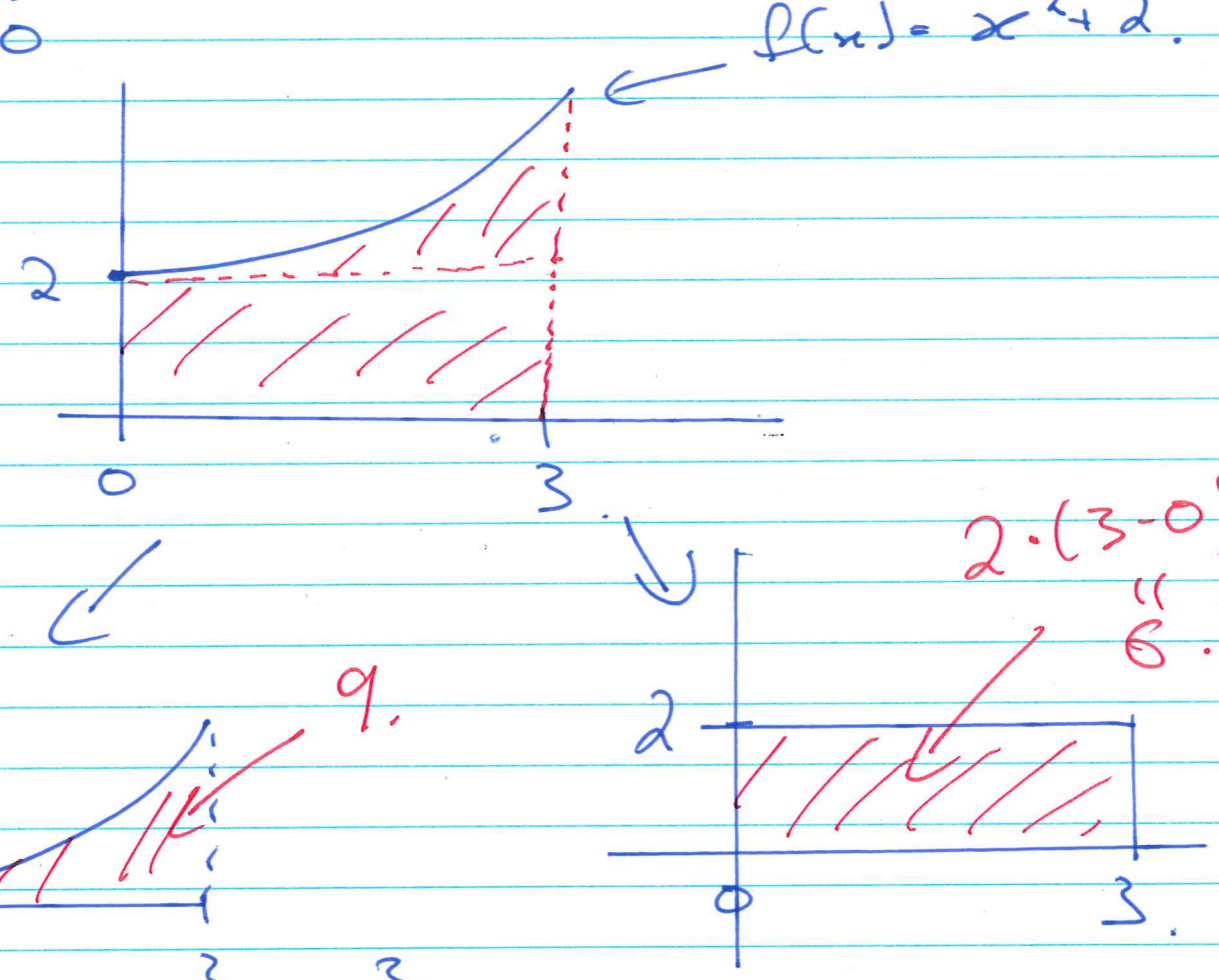
(k - constant).

$$\int_a^b k f(x)\,dx.$$

$$= k\int_a^b f(x)\,dx.$$

(6)

A guess for: $\int_0^3 (x^2 + 2)\,dx$? $= 15$.



We write: $\int_0^3 (x^2 + 2)\,dx$

$= \int_0^3 x^2\,dx + \int_0^3 2\,dx$.

In general,

$$\int_a^b \{f(x) \pm g(x)\}\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$$

But how do we actually compute

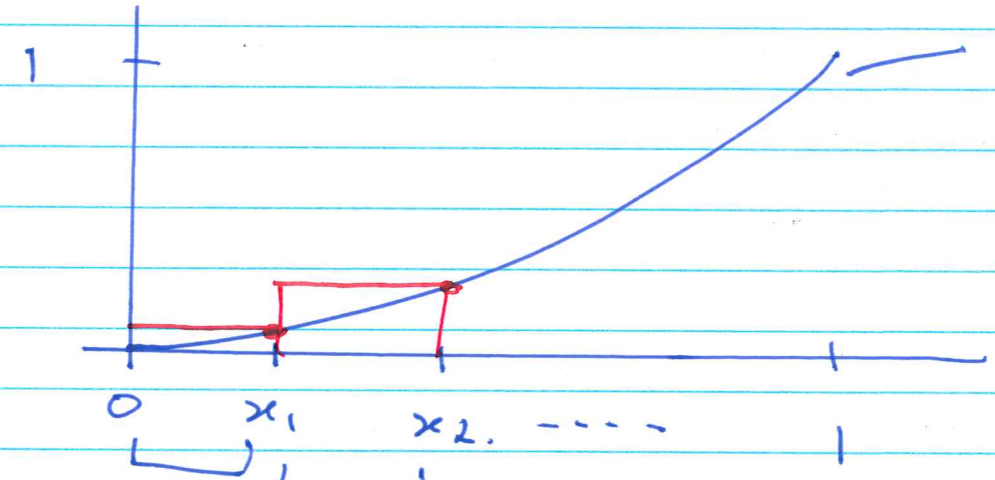
$$\int_0^3 x^2\,dx = ? \;?$$

The hard way: (akin to definition of the derivative).

$$\int_0^1 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x.$$

one can compute this limit.

What is
- Δx
- x_i
- $f(x_i)$.



$\Delta x = 1/n$.

$x_1 = 1/n$. $\quad x_3 = 3/n$.

$x_2 = 2/n$ $\quad x_i = i/n$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \frac{1}{n}.$$

= ..... = 1/3.

↑ lots of math.