1

Nov. 7.

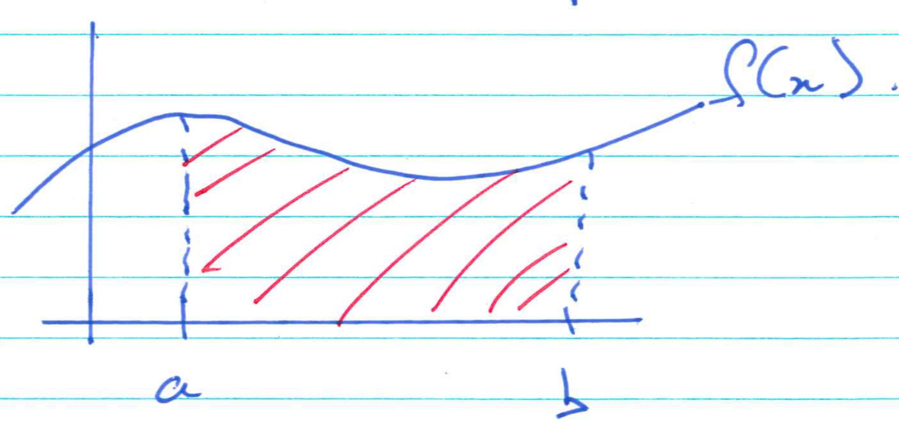
Midterm
- grades posted. (add 1 to total).
- back in Labs
Average : 62.9%
Median : 65.7%.

- Quiz #4 - Monday
- HW8 Due Monday (posted today).

Definite Integrals:

Last class we defined the definite integral as:

$$\int_a^b f(x)\,dx = \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x}^{\text{Computing this limit is impractical.}}.$$



Fundamental Theorem of Calculus.

There are a few parts to FTC. Basically, it says that derivatives and integrals are inverse/oposite operations.

We will need the following result:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F'(x) = f(x)$.

We call $F(x)$ an anti-derivative.

Let's consider a simple integral.

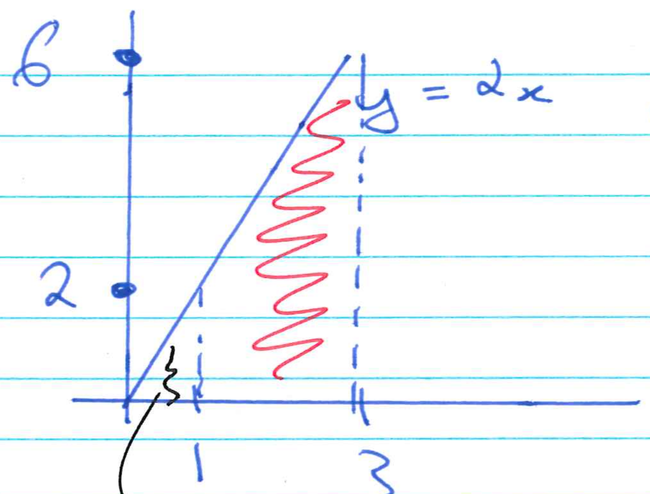
Compute: $\int_1^3 2x\,dx$.

1) using area of a triangle.

2) using FTC.

③

1)



$\frac{1 \cdot 2}{2} = 1.$

whole triangle.

$\frac{3 \cdot 6}{2} = 9.$

$$\int_1^3 2x\,dx = 9 - 1 = 8$$

2) Find $\left(\begin{array}{l} F(x) = x^2. \\ F'(x) = 2x = f(x). \end{array} \right)$.

So,

$$\int_1^3 2x\,dx = F(3) - F(1) = 3^2 - 1^2 = 9 - 1 = 8.$$

Some notation:

$$\int_1^3 2x\,dx = \boxed{x^2 \Big|_{x=1}^{3}} = 3^2 - 1^2 = 8.$$

Examples:

- $\int_0^1 x^2\,dx$ $\qquad f(x) = x^2$, $F(x) = \frac{x^3}{3}$, $\mathcal{Z}x$.

$$= \frac{x^3}{3}\Big|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = 1/3.$$

- $\int_1^2 \left(\sqrt{x} + 3x^4 - \frac{1}{3\sqrt{x^2}}\right)dx.$

$$f(x) = x^{1/2} + 3x^4 - x^{-2/3}.$$

$$F(x) = \frac{2}{3}x^{3/2} + \frac{3}{5}x^5 - 3x^{1/3}.$$

(aside: $\left(\frac{2}{3}x^{3/2}\right)' = \frac{2}{3}\cdot\frac{3}{2}x^{1/2} = x^{1/2}$).

In general:

$$\left(\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1\right).$$

⑤

$$\int_1^2 \left(\sqrt{x} + 3x^4 - \frac{1}{3\sqrt[3]{x^2}} \right) dx$$

$$= \frac{2}{3}x^{3/2} + \frac{3}{5}x^5 - 3x^{1/3} \Big|_1^2 .$$

$$= \frac{2}{3}(2)^{3/2} + \frac{3}{5}(2)^5 - 3(2)^{1/3}$$
$$- \frac{2}{3}(1)^{3/2} - \frac{3}{5}(1)^5 + 3(1)^{1/3} .$$

What if $n = -1$?

$$\int_1^2 \frac{1}{x}\, dx = \int_1^2 x^{-1}\, dx .$$

The anti-derivative of $\frac{1}{x}$ is $\ln x$.

$$\int_1^2 \frac{1}{x}\, dx = \ln x \Big|_1^2 = \ln(2) - \ln(1)$$
$$= \ln(2) .$$

⑥

Example: $\displaystyle\int_0^{\pi} \sin x \, dx$.

$f(x) = \sin x$

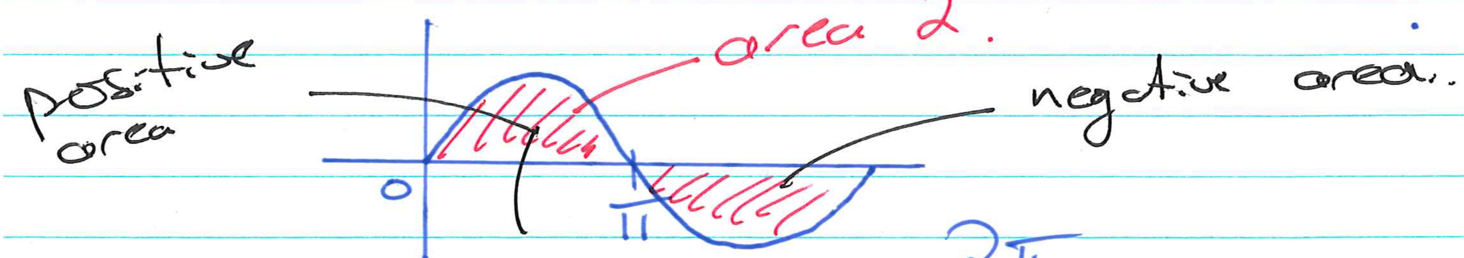
$F(x) = -\cos x$. $\quad \left((-\cos x)' = -(-\sin x) = \sin x \right)$.

$$\int_0^{\pi} \sin x \, dx = (-\cos x) \Big|_0^{\pi}.$$

$$= -\cos(\pi) - (-\cos(0)).$$

$$= 1 + 1$$

$$= 2.$$



Clicker Q! What is $\displaystyle\int_0^{2\pi} \sin x \, dx$?

→ A) 0 $\qquad$ B) 1

C) 2 $\qquad$ D) 4.

(7).

$$\int_0^{2\pi} \sin x \, dx = -\cos x \Big|_0^{2\pi}$$

$$= -\cos(2\pi) - (-\cos(0))$$

$$= -1 + 1$$

$$= 0$$

- $$\int_0^1 e^x \, dx = e^x \Big|_0^1 = e^1 - e^0$$
$$= e^1 - 1$$

- $$\int_0^1 e^{-x} \, dx = -e^{-x} \Big|_0^1$$

$$\left(\text{check: } (-e^{-x})' = -e^{-x}(-1) = e^{-x} \right)$$