1

Oct. 14.

- HW4 solutions posted.
- HW5 Due Monday
- Quiz #3 Oct. 21 (Friday)
  - limit definition – Conceptual
    – Computational.
  - power, product rule.
- <u>Midterm</u>: Oct. 31

- practice problems updated.
- Observe ~~learning~~ Learning Objectives.

$$\frac{d}{dx}\left(x^n\right) = n x^{n-1}.$$

<u>Ex</u>: $\left(x^{3/2}\right)' = \frac{3}{2} x^{1/2}.$

Clicker Q: Can we use power rule to take the derivative of

$$f(x) = 2^x ?$$

A) Yes
→ B) No
C) Don't know.

↑ 2^x is an exponential function and not a polynomial.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$= \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h}.$$

a bunch of algebra.
$$= \vdots$$

$$= 2^x \ln(2).$$

In general, if $f(x) = a^x$

$$f'(x) = a^x \ln(a).$$

3

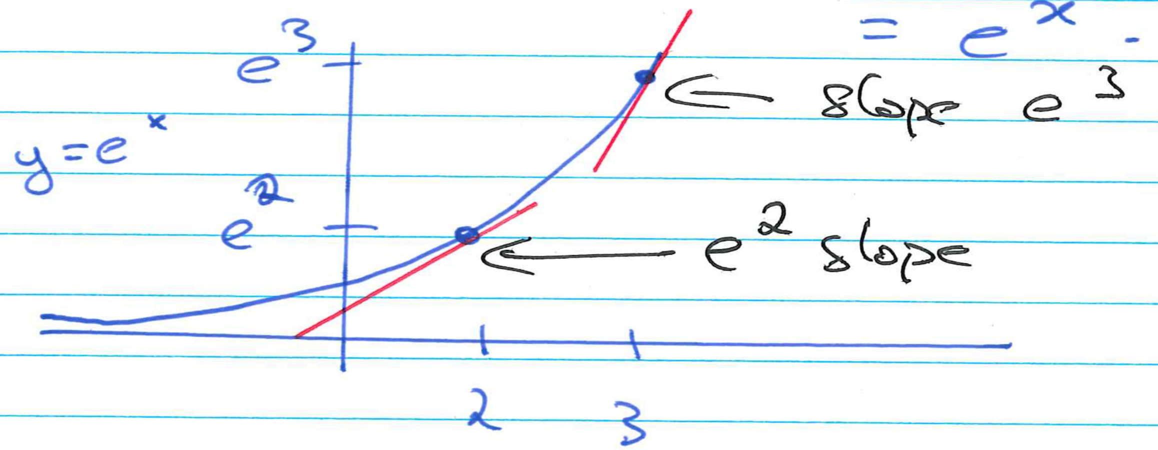
In particular,

this function is its own derivative.

$$f(x) = e^x$$

$$f'(x) = e^x \ln(e).$$

$$= e^x.$$

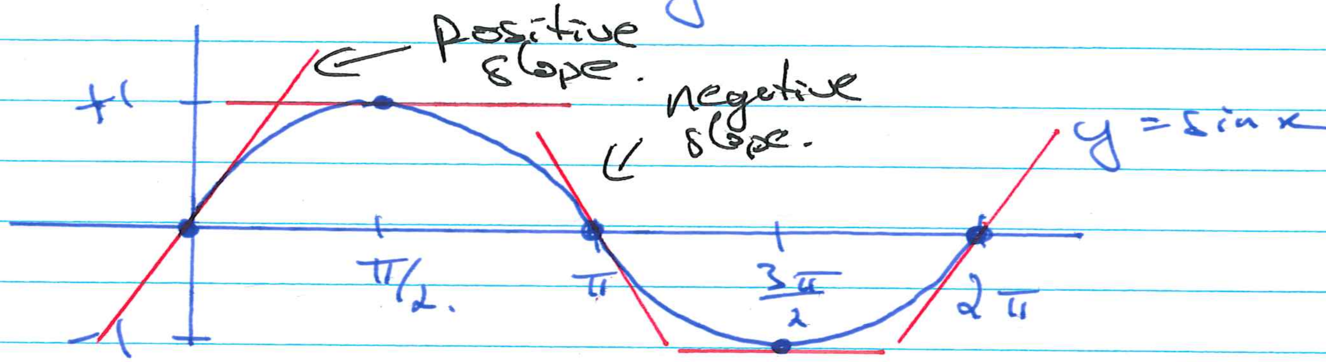


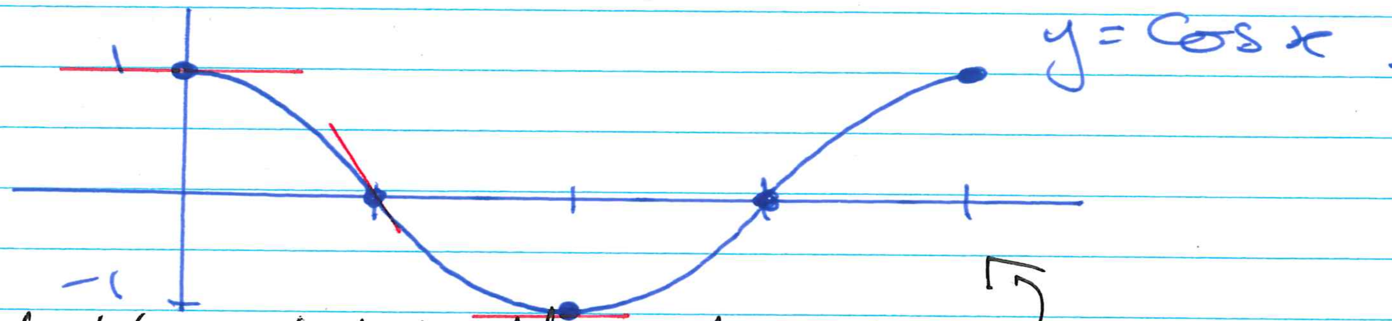
Slope at $x = 2$ is

$$f'(2) = e^2.$$

this is why e^x is so important.

What about trig. functions?





Let's sketch the derivative

We suspect that $\frac{d}{dx}(\sin x) = \cos x$.

This can be proved using the definition.

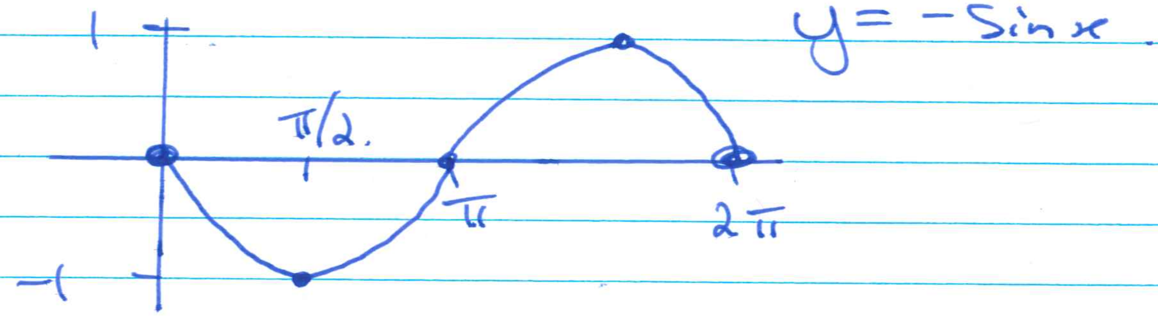
$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}.$$

lots of trig $\Bigg\{ \quad = \vdots$

$$= \cos x.$$

(5)

We can also sketch the derivative of cosine :



$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

By the way this is only true if we use radians.

Clickers①: What is the derivative of $f(x) = 3\sin x$?

A) 3 → C) $3\cos x$
B) 0 D) $\cos x$

constant.

Note: $\frac{d}{dx}\left(c\, f(x)\right) = c \cdot f'(x)$.

Recall! $(f+g)' = f' + g'$

$(f-g)' = f' - g'$.

Clicker Q: Is $(x \cdot \sin x)' = 1 \cdot \cos x$?

A) Yes
→ B) No
→ C) I don't know.
because we haven't learned product rule yet.

The **Product** **Rule**: (§2.2).

$$(f \cdot g)' = f'g + fg'.$$

A Simpler **example**: $h(x) = (x+1)(x-2)$.

$h(x) = x^2 - 2x + x - 2$
$= x^2 - x - 2$.

power rule.

$h'(x) = 2x - 1 + 0 = 2x - 1$.

Now product rule.

$h(x) = \underbrace{(x+1)}_{f(x)}\underbrace{(x-2)}_{g(x)}$.

$f(x) = x+1$, $g(x) = x-2$

$f'(x) = 1$, $g'(x) = 1$.

So, $h'(x) = f'(x)g(x) + g'(x)f(x)$.

product rule. →

$= 1 \cdot (x-2) + 1 \cdot (x+1)$.

$= x - 2 + x + 1$

$= 2x - 1$ ← as expected.

Examples: Find the derivative of each.

1) $x\sin x$

2) $7\sin x \cos x$

3) xe^x.

(aside: $(x)'$
$= (x^1)'$
$= (1 \cdot x^0) = 1 \cdot 1$
$= 1$.)

1) $\underbrace{x}_{f}\underbrace{\sin x}_{g}$ $f(x) = x$ $g(x) = \sin x$

$f'(x) = 1$ $g'(x) = \cos x$.

$(x\sin x)'$

$= 1 \cdot \sin x + x\cos x$

$= \sin x + x\cos x$.

//.