- HW6 Due Today
- Labs for review this week
- Wed/Fri Class Review
- No HW this week
- See Lab 7 for Midterm Practice
- Last year's midterm is posted online.

## Office Hours This Week

Wed: 3-4:30pm (↑ Me) in LSK 300B (↑ near MLC)

Thurs: 3:30-4:30pm (↑ Me and Edward) in ~~TBA~~ GEOG 201.

Fri: 2-4pm (↑ Me) 4-5pm (↑ Edward) in MATX 1118 (↑ Math Annex.)

Topics:

- Functions (trig. exp. log.)
- Limits
- Asymptotes
- Tangent Lines / Limit Def.
- Derivative Rules
  - $e^x$ $\sin x$ $\cos x$ $\ln x$
  - power, product, quotient, chain.

## Where to find Questions:

- Examples done in class
- HW problems
- Lab problems
- Quiz problems
- Practice Problems
- Review Materials
- Check Learning Goals
- Last Year's Quizzes, HWs.

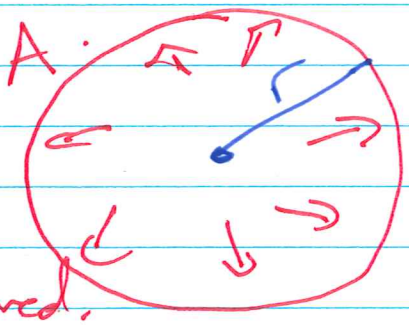
(3)

Our last differential calculus topic is Related Rates.

## Related Rates (§2.6)

Example: The radius of a circular forest fire is increasing at a constant rate of 5 km/h. How fast is the area consumed by the fire increasing when the radius is 10 km?

1. Picture / Variables.




A - Area
r - radius.
t - time.

2. Given/Required.

- $\frac{dr}{dt} = 5 \text{ km/h}$ ← Given Rate.
- $\frac{dA}{dt} = ?$ → Required Rate.

Want to relate these rates.

(4)

3. Equation. $A = \pi r^2$. ← equation of circle.

$A$ is a function of time.
$r$ is a function of time.

We would like to introduce the derivatives. Take the derivative of both sides with respect to time.

4. Chain Rule. $\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$.

$$\frac{dA}{dt} = \pi \frac{d}{dt}(r^2).$$

$$= \pi \cdot 2r \cdot \frac{dr}{dt}.$$

$$= 2\pi r \frac{dr}{dt}.$$

aside:

$r$ is variable. → $\frac{d}{dr}(r^2) = 2r$

$t$ is variable → $\frac{d}{dt}(r^2) = 2r \cdot \frac{dr}{dt}$.

(8)

$$\frac{d}{dt}\overbrace{(t^3+4)}^{r}{}^2 \cdot \underbrace{\frac{dr}{dt}}$$

$$2\underbrace{(t^3+4)}_{r} \cdot 3t^2 \cdot$$

We have $\frac{dr}{dt} = 5 \text{ km/h}$.

$r = 10 \text{ km}$.

5. Substitute / Solve.

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

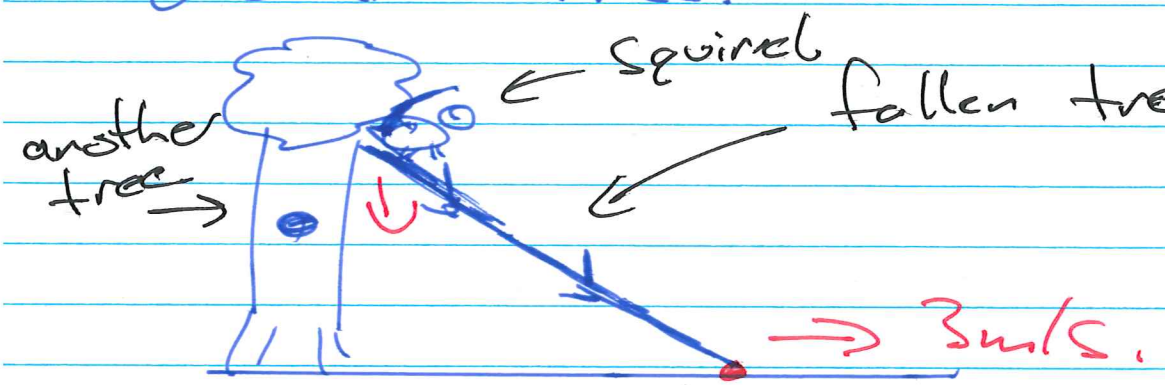
$$= 2\pi(10 \text{ km}) \cdot (5 \text{ km/h})$$

$$= 100\pi \text{ km}^2/\text{h}$$

This is how fast the area is increasing.

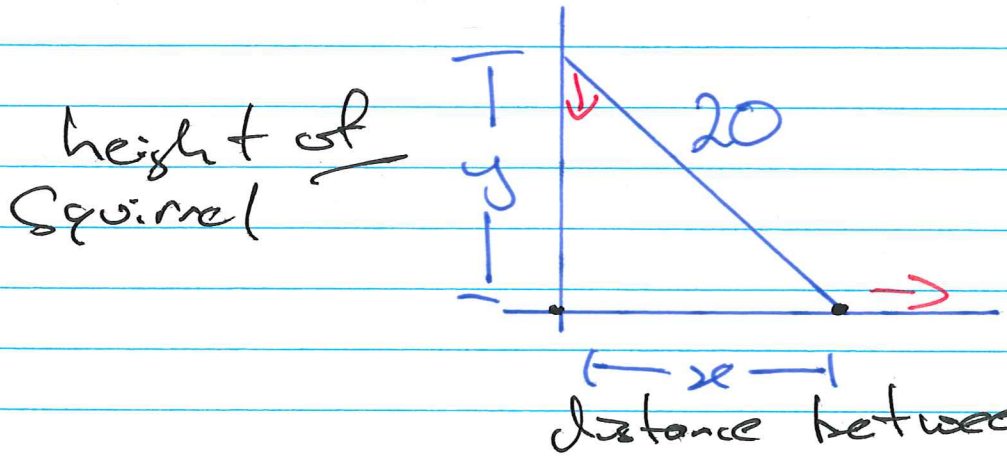
③

Example: A 20m high fallen tree is resting against another tree.




The base slides at a rate of 3m/s. How fast is the squirrel descending when the base of the fallen tree is 12m from the other.

1. Picture / Notation.



① 2. Given / Required.

$$\begin{cases} \frac{dx}{dt} = 3 \text{ m/s} \\ \frac{dy}{dt} = ? \quad \text{when} \quad x = 12 \text{ m} \end{cases}$$

3. Equation.

$$x^2 + y^2 = 20^2.$$

4. Chain Rule.

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(20^2)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0.$$

(have. want.)

5. Solve / Substitute:

$$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$$

(8)

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}.$$

$$= \frac{-(12m)}{(16m)} \cdot 3\,m/s.$$

Find $y$ using, $x^2 + y^2 = 20^2$.

$$y^2 = 20^2 - 12^2$$

$$y = \sqrt{20^2 - 12^2}.$$

$$y = 16.$$

$$\frac{dy}{dx} = -\frac{12}{16} \cdot 3\ m/s.$$

$$= -9/4\ m/s.$$