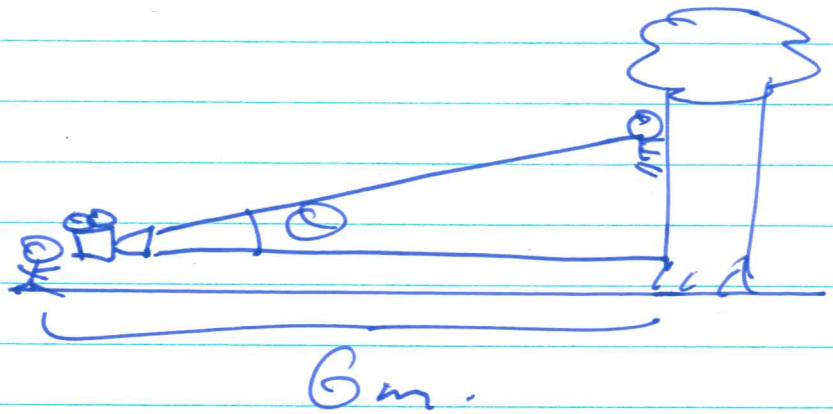
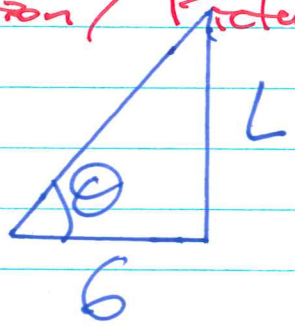
- Midterm: Monday.

One more Related Rates problem.

Your lumberjack friend is climbing a tree at a constant speed of 0.5 m/s. You stand 6 meters from the tree and film with your camera. How fast is the camera's angle changing when your friend is 6m up.



1. Notation / Picture.



θ and L are functions of time, t.

(2)

2. Speed L is changing in time.

Given Rate: $\frac{dL}{dt} = 1/2$ m/s.

Required Rate: $\frac{d\theta}{dt} = ?$ when $L = 6$ m.

3. Equation. $\tan\theta = \frac{L}{6}$.

4. Chain Rule. Let's take the derivative of both sides with respect to time.

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{L}{6}\right).$$

$$= \frac{1}{6}\frac{d}{dt}(L).$$

$$= \frac{1}{6}\frac{dL}{dt}.$$

(3) θ is like the inside. $\longrightarrow$ $g(t)$.

What about $\frac{d}{dt}\left(\underbrace{\tan\theta}_{\tan x}\right)$.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$.

(aside: $(\tan x)' = \left(\frac{\sin x}{\cos x}\right)'$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$.)

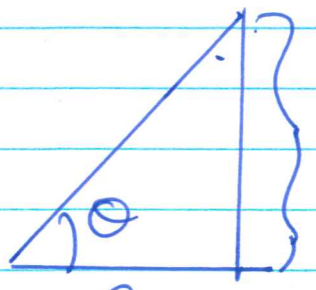
$\frac{d}{dt}(\tan\theta) = \frac{1}{\cos^2\theta} \cdot \frac{d\theta}{dt}$ ← want.

So $\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{1}{6}\frac{dL}{dt}$ ← 0.5.

5. Substitute/Solve

$\frac{d\theta}{dt} = \frac{1}{6}\cos^2\theta \frac{dL}{dt}$ ← 0.5.

(4)



Want $\frac{d\theta}{dt}$ when $L = 6$.

$\Rightarrow \theta = \pi/4$.

$$\frac{d\theta}{dt} = \frac{1}{6}\cos^2\left(\frac{\pi}{4}\right) \cdot \frac{1}{2}.$$

$$= \frac{1}{6}\left(\frac{1}{\sqrt{2}}\right)^2 \cdot \frac{1}{2}$$

$$= \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{2} =$$

$$= \frac{1}{24} \text{ radians / second.}$$

$$= 2.4 \text{ deg/sec.}$$

Review: Derivatives you need to know:

$$\{f(x) \pm g(x)\}' = f'(x) \pm g'(x).$$

constant.

$$(cf(x))' = c f'(x).$$

$$\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x).$$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}.$$

$$\{f(g(x))\}' = f'(g(x)) \cdot g'(x).$$

$$\{x^n\}' = nx^{n-1}.$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x.$$

$$(e^x)' = e^x, \quad (\ln x)' = 1/x.$$

(6)

Double Chain Rule:

$$\left\{ \sin\left(\underbrace{e^{2x}}_{\text{first. inside}} \right) \right\}' \qquad \text{second inside: } e^{2x}$$

$$= \cos\left(e^{2x}\right) \cdot \left(e^{2x}\right)'$$

$$= \cos\left(e^{2x}\right) \cdot e^{2x} \cdot 2 \quad \rightarrow \text{derivative of } 2x.$$

$$= 2e^{2x} \cos\left(e^{2x}\right).$$

$$= \cos\left(e^{2x}\right) \cdot e^{2x} \cdot (2x)'.$$

(4)

1) $\cos(\ln(\sin x))$

2) $\sqrt{\cos(\ln(\sin x))}$.

1) $-\sin(\ln(\sin x)) \cdot (\ln(\sin x))'$

$= -\sin(\ln(\sin x)) \cdot \frac{1}{\sin x} \cdot \cos x$.

$= \frac{-\sin(\ln(\sin x))\cos x}{\sin x}$.

2) $\frac{1}{2}(\cos(\ln(\sin x)))^{-1/2} \cdot$

$\left(-\sin(\ln(\sin x)) \frac{\cos x}{\sin x}\right)$.