Sept. 12

Composition of Piecewise Functions: Note

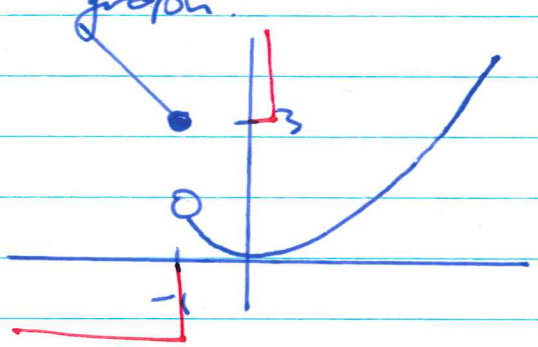
Consider $$f(x) = \begin{cases} 2x-1, & x \geq 3 \\ -x+4, & x < 3 \end{cases}$$

$$g(x) = \begin{cases} x^2, & x > -1 \\ -3x, & x \leq -1 \end{cases}.$$

from class. Let us try to find a formula for $f(g(x))$. We consider several cases depending on the value of input x.

For $x \leq -1$ we have that $g(x) = -3x \geq 3$. Observe the graph.

$y = g(x)$:



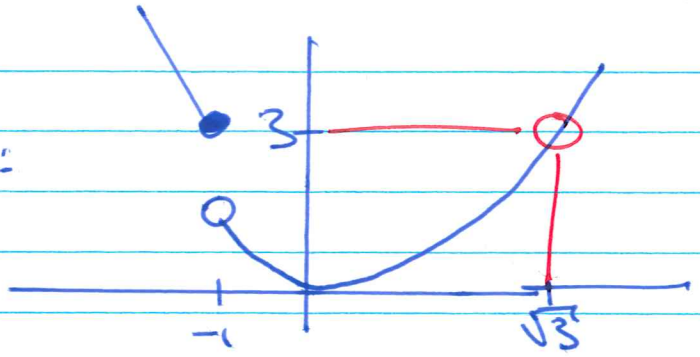
In this way for $x \leq -1$ we always take the first branch of f. That is:

$$f(g(x)) = f(-3x) = 2(-3x) - 1 = -6x - 1, \quad x \leq -1.$$

Take now $x > -1$. The value $g(x) = x^2$ can be less than 3 or bigger than 3. For this reason we need two cases. Observe again the graph to find the cutoff.

(2)

$y = g(x)$:



For $-1 < x < \sqrt{3}$ we have $g(x) < 3$ and so we take the second branch of f.

$$f(g(x)) = f(x^2) = -(x^2) + 4 = -x^2 + 4, \quad -1 < x < \sqrt{3}.$$

For $x \geq \sqrt{3}$ we have $g(x) \geq 3$ and so we take the first branch of f:

$$f(g(x)) = f(x^2) = 2(x^2) - 1 = 2x^2 - 1, \quad x \geq \sqrt{3}.$$

Putting everything together yields

$$f(g(x)) = \begin{cases} -6x - 1, & x \leq -1 \\ -x^2 + 4, & -1 < x < \sqrt{3} \\ 2x^2 - 1, & x \geq \sqrt{3} \end{cases}.$$