Sept. 21

- HW1 Solutions Posted
  - look at them
- HW2 due Monday
  - typo in #3 is corrected.
- Quiz 1 Friday

Office Hours: Friday 2-3:30pm.

Simplify: $\frac{e^{x^3} \cdot e^{2x^2} \cdot e^{x}}{e^3}$

$$= \frac{e^{x^3+2x^2+x}}{e^3}$$

$$= e^{x^3+2x^2+x} \cdot e^{-3}$$

$$= e^{x^3+2x^2+x-3}$$

Let $2^x = 8$
what is x? $\Rightarrow$ $x = 3$.

What about $e^x = 2$?
We need logarithms!

Logarithms and Logarithmic Functions.

Let $2^y = 8$.

We write $y = \log_2(8) = \log_2 8$.

(in this example we say $\log_2(8) = 3$).

In general,

if $x = b^y$ then $y = \log_b x$.

Example: Find $y = \log_3(9)$.

i.e. $3^y = 9$. So, $y = 2$.

Note,

$$\log_3(9) = \log_3(3^2) = 2.$$

The function $g(x) = \log_b(x)$ is the inverse function of $f(x) = b^x$.

That is

$$g(f(x)) = \log_b(b^x) = x.$$

($f(x)$: input of x; x: output of x.)

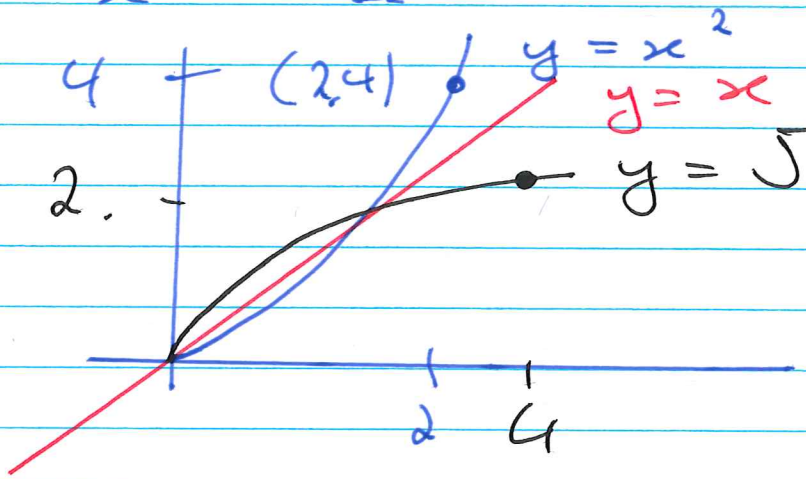
Do you know any other inverses?

$f(x) = x$
$g(x) = 1/x$

$g(f(x)) = g(x) = 1/x$.

- x^2 and $\sqrt{x}$.

$x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$
$x = 3$.

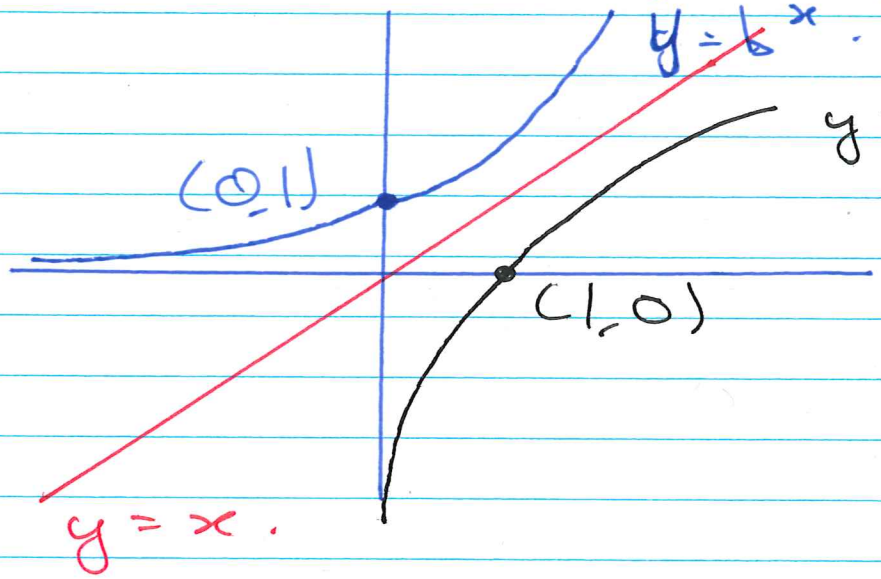


The point $(2,4)$ is on x^2.
The point $(4,2)$ is on $\sqrt{x}$.

(4)

Now let's draw $y = b^x$ $(b > 0)$.

and $y = \log_b x$.



What is the domain of $f(x) = \log_b(x)$?

- $\{x \in \mathbb{R} : x > 0\}$.
- $(0, \infty)$

Range of $\log_b(x)$?

- $\{x \in \mathbb{R}\}$, - $(-\infty, \infty)$.

$\{x \in \mathbb{R}\}$ or $\{y \in \mathbb{R}\}$.

$\{x \in \mathbb{R}\} = \{y \in \mathbb{R}\} = \{\cdot : \cdot \in \mathbb{R}\}$.

Domain of $f(x) = b^x$.

Range of $\log_b x$ → • $\{x \in \mathbb{R}\}$, $(-\infty, \infty)$

Range of $f(x) = b^x$.

- $\{x \in \mathbb{R} : x > 0\}$, $(0, \infty)$.
- $\{y \in \mathbb{R} : y > 0\}$.
- $\{\ddot{\smile} \in \mathbb{R} : \ddot{\smile} > 0\}$

Domain of $\log_b x$ →

Especially important for us:

$$f(x) = \log_e x = \ln x.$$

the so called natural logarithm.

Example: Solve for x:

$$e^{x^2+1} = 5.$$

$$\ln(e^{x^2+1}) = \ln 5 = \log_e 5.$$

$$x^2 + 1 = \ln 5.$$

$$x^2 = \ln 5 - 1$$

$$x = \pm\sqrt{\ln 5 - 1}.$$

(20)

To work with logarithms we need the following identities:

1) $\log_b(xy) = \log_b x + \log_b y$.

2) $\log_b(x/y) = \log_b x - \log_b y$.

3) $\log_b(x^p) = p \log_b x$.

Simplify: $\ln\left(\frac{1}{\sqrt{3x+1}}\right)$.