① Sept. 28

<u>Limits</u>: But how do we compute limits given an equation?

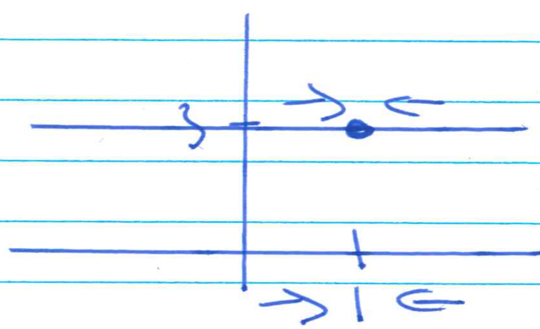
Last class we computed limits using graphs (§ 1.1)

Today, equations (§ 1.2).

<u>Clicker Q</u>: What is $\lim_{x \to 1}(3)$

A) 0 C) 2 E) D.N.E.

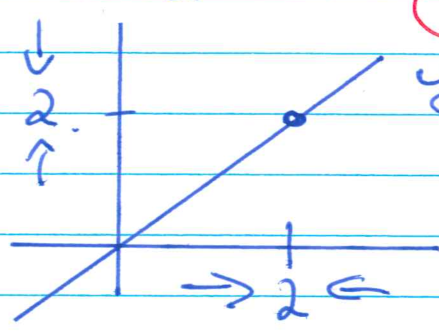
B) 1 → D) 3



$y = 3$

<u>Clicker Q</u>: $\lim_{x \to 2} x$. Some options.

C.



$y = x$.

$\lim_{x \to 2} x = 2$.

Using the above simple limits and the following limit laws we can compute limits of "nice" functions.

Limit Laws (Start of § 1.2).

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$.

then

a) $\lim_{x \to a} \left(f(x) + g(x) \right) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = M + L$

b) $\lim_{x \to a} \left(f(x) - g(x) \right) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$

constant.

c) $\lim_{x \to a} \left[k f(x) \right] = k \lim_{x \to a} f(x) = kL$

d) $\lim_{x \to a} \left[f(x) g(x) \right] = \left\{ \lim_{x \to a} f(x) \right\} \left\{ \lim_{x \to a} g(x) \right\} = M \cdot L$

e) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = L/M \quad (M \neq 0)$

f) $\lim_{x \to a} \left[\left(f(x) \right)^n \right] = \left(\lim_{x \to a} f(x) \right)^n = L^n$.

Examples: Find $\lim_{x\to -2}\left(\dfrac{x^2-3x+2}{x^3+6}\right)$.

$$= \frac{\lim_{x\to -2}(x^2-3x+2)}{\lim_{x\to -2}(x^3+6)}$$

(e)

(a), (b) (f) (c)

$$= \frac{\lim_{x\to -2}(x^2) - \lim_{x\to -2}(3x) + \lim_{x\to -2} 2}{\lim_{x\to -2}(x^3) + \lim_{x\to -2} 6}$$

(f)

$$= \frac{\left(\lim_{x\to -2} x\right)^2 - 3\lim_{x\to -2} x + \lim_{x\to -2} 2}{\left(\lim_{x\to -2} x\right)^3 + \lim_{x\to -2} 6}$$

$$= \frac{(-2)^2 - 3(-2) + 2}{(-2)^3 + 6}.$$

$$= \frac{4+6+2}{-8+6} = \frac{12}{-2} = -6.$$

Notice that really all we did was plug in $x = -2$.

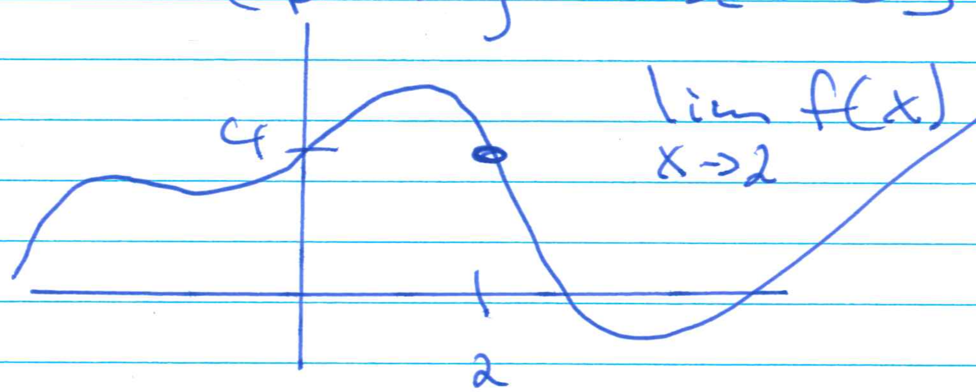
Observe.

$$\lim_{x \to -2} \frac{x^2 - 3x + 2}{x^3 + 6} = \frac{(-2)^2 - 3(-2) + 2}{(-2)^3 + 6}.$$
$$= -6.$$

If you have a "nice" function to compute a limit you need only substitute.

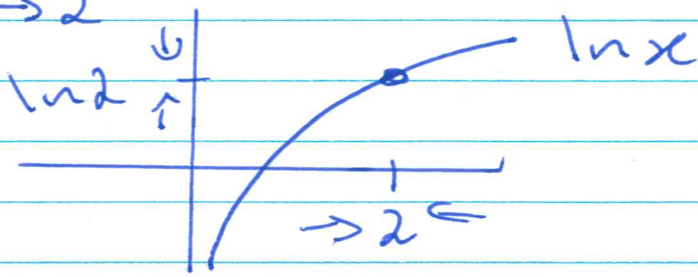
Nice functions include:

- polynomials (constants).
- rational functions of polynomials.
  - with no zero in the denominator.
- $\sin x$ $\cos x$ e^x
- $\sqrt{x}$ and $\log x$ (providing $x > 0$).



$$\lim_{x \to 2} f(x) = 4.$$

Example: $\lim_{x \to 2} \ln x = \ln 2.$



Substitution won't always work.

Example: $\lim_{x \to 3} \underbrace{\frac{x^2 - 2x - 3}{x - 3}}_{f(x)}$

If we try to plug in $x = 3$:

$$f(3) = \frac{3^2 - 2 \cdot 3 - 3}{3 - 3} = \text{"}\frac{0}{0}\text{"}.$$

not a number.

$f(3)$ does not exist
but the limit might still exist.

Let us factor

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}}$$

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(We can cancel the $(x-3)$'s since x is just close to 3 and not equal to 3)

substitution.

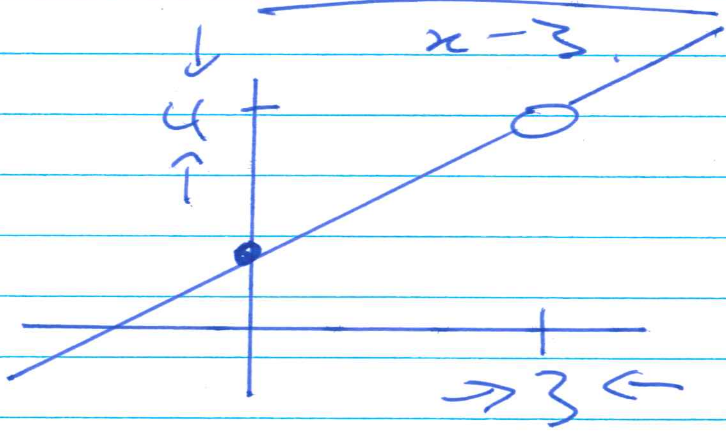
$$= \lim_{x \to 3} x+1 \quad = 3+1 \quad = 4.$$

after we substitute we have taken the limit (and so stop writing lim).

$$f(x) = \begin{cases} x+1, & x \neq 3 \\ \text{undefined}, & x = 3 \end{cases}$$

Same.

$$= \frac{x^2 - 2x - 3}{x - 3}.$$



Example: Find $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$

and $\lim_{x \to 2} f(x)$

where $f(x) = \begin{cases} x+3, & x \geq 2 \\ -x^2-1, & x < 2. \end{cases}$

- $\lim_{x \to 2^-} f(x)$

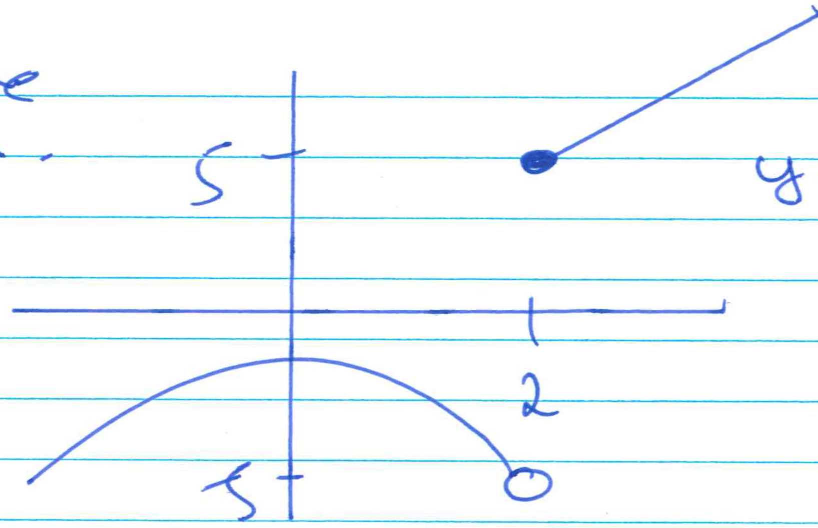
$$= \lim_{x \to 2^-} (-x^2-1) = -2^2-1 = -4-1 = -5.$$

- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x+3) = 5.$

- $\lim_{x \to 2} f(x)$ = D.N.E. since the two one sided limits are not equal.

(8)

Note the graph.



$y = f(x)$.