Math 190 Quiz 1: Friday September 23

The quiz is 10 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit.

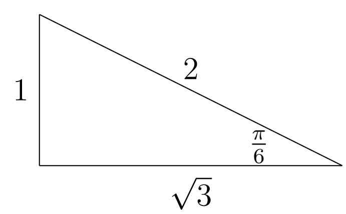
Questions:

1. Find all x in $[0, 2\pi)$ so that

$$\cos x = \frac{\sqrt{3}}{2}.$$

Support your answer with the relevant part of the unit circle and the appropriate special triangle.

Solution: Consider the following special triangle



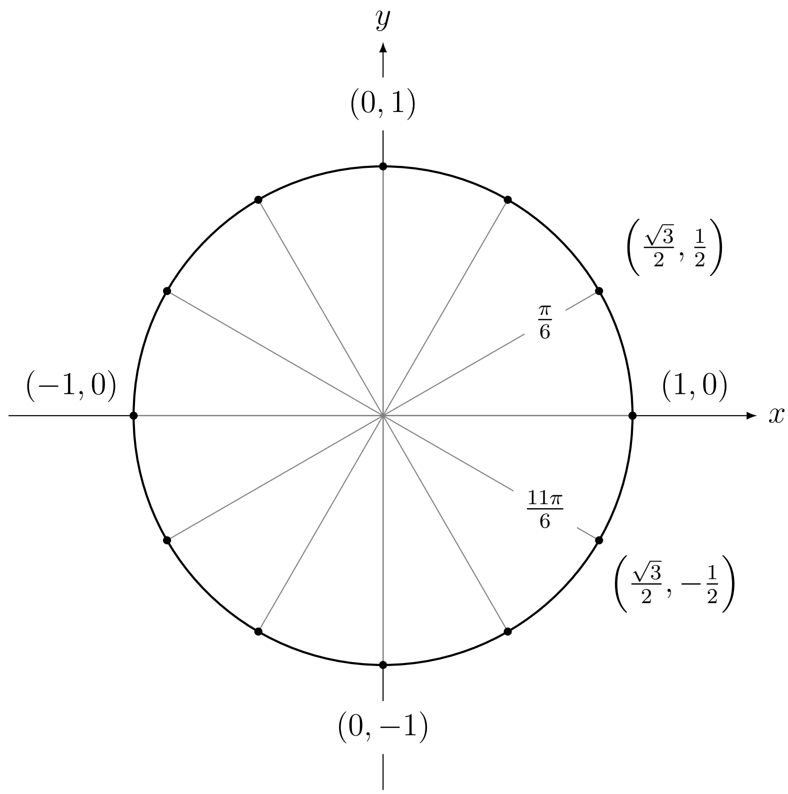
We see that

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

and so our first solution is $x = \pi/6$. To find the other solution we consult the unit circle to see that

$$\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

And so our two solutions are $x = \pi/6$ and $x = 11\pi/6$.



2. Consider the functions

$$f(x) = \begin{cases} 5x, & x \geq 2 \\ 3x, & x < 2 \end{cases}$$
$$g(x) = x^2 + 4.$$

Find a formula for the composition $f(g(x))$.

Solution: To compose these functions we must think about the possible outputs from $g(x)$. For x values such that $g(x) \geq 2$ we would select the first branch. For x values where $g(x) < 2$ we would select the second branch. This could be denoted in the following way

$$f(g(x)) = \begin{cases} 5(x^2+4), & x^2+4 \geq 2 \\ 4(x^2+4), & x^2+4 < 2 \end{cases}.$$

However, inspecting the range of $g(x)$ we can observe that $g(x) = x^2 + 4 \geq 4 > 2$ and so no matter the value for x we will always have $g(x) > 2$. In this way we can only ever access the first branch. Therefore

$$f(g(x)) = 5(x^2 + 4).$$