Math 190 Quiz 2: Solutions

The quiz is 10 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit. When asked to compute a limit: if the limit exists find its value, if the limit does not exists but 'equals $\pm\infty$' say so, otherwise explain why the limit does not exist.

Questions:

1. Compute $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$.

   Solution: To compute this limit we first multiply by the conjugate to eliminate the square root in the numerator. We note that immediate substitution results in '0/0.' There follows

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} &= \lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} \\
&= \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)} \\
&= \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)} \\
&= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} \\
&= \frac{1}{\sqrt{4}+2} \\
&= \frac{1}{2+2} \\
&= \frac{1}{4}.
\end{aligned}$$

   And so the limit exists and takes the above value.

2. Compute $\lim_{x\to 1} f(x)$ where

$$f(x) = \begin{cases} x^2 - 2x + 1, & x < 1 \\ 7, & x = 1 \\ \ln x, & x > 1 \end{cases}.$$

   Solution 1: To investigate this limit let us first compute the one sided limits. First consider $x > 1$

$$\begin{aligned}
\lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} \ln x \\
&= \ln(1) \\
&= 0
\end{aligned}$$

where we have selected the third branch of $f(x)$. Now for $x < 1$ we select the first branch. Observe

$$\begin{aligned}\lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} x^2 - 2x + 1 \\ &= 1^2 - 2(1) + 1 \\ &= 0.\end{aligned}$$

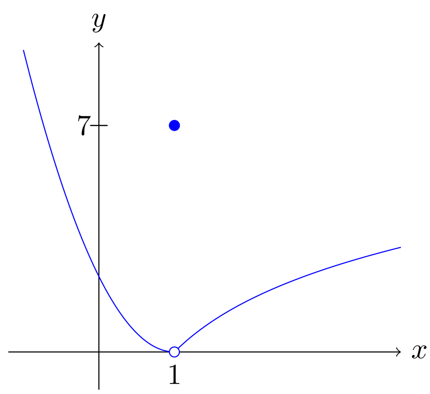
Having computed the two one sided limits and achieved the same value for both we conclude that the limit does in fact exist and takes the value 0. That is

$$\lim_{x\to 1} f(x) = 0.$$

Solution 2: Alternatively we can draw the graph of this function. We recall the graph of $\ln x$ to plot $f(x)$ when $x > 1$. When $x < 1$ we can factor the relevant branch to see

$$x^2 - 2x + 1 = (x - 1)^2$$

which is our familiar parabola x^2 just shifted one to the right. Behold the graph of $f(x)$



From the sketch of the graph it is clear that $\lim_{x\to 1} f(x) = 0$.