Math 190 Quiz 3: Solutions

The quiz is 10 minutes long and has two questions. No calculators or other aids are permitted. Show all of your work for full credit.

Questions:

1. Compute the derivative of $5\sqrt[4]{x} + x \cdot x + e^{\pi}$ using any method.
   Solution: Let $f(x) = 5\sqrt[4]{x} + x \cdot x + e^{\pi}$. We can rewrite our function as
   $$f(x) = 5x^{1/4} + x^2 + e^{\pi}.$$
   In this way we compute the derivative using power rule
   $$\begin{aligned} f'(x) &= \frac{5}{4}x^{-3/4} + 2x + 0 \\ &= \frac{5}{4}x^{-3/4} + 2x. \end{aligned}$$
   Note we could have used power rule to compute the derivative of $x \cdot x$ to the same end.

2. Show that the derivative of x^2 is $2x$ by using the definition of the derivative (no credit will be given for using a different method).
   Solution: We first recall the definition of the derivative
   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$
   We let $f(x) = x^2$ and substitute to see
   $$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \to 0} \frac{h(2x+h)}{h} \\ &= \lim_{h \to 0} 2x + h \\ &= 2x + 0 \\ &= 2x \end{aligned}$$
   where we have computed the limit. Therefore $(x^2)' = 2x$ as required.