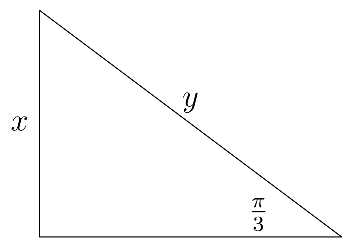
Math 190 Quiz 4: Solutions

The quiz is 15 minutes long and has one question. No calculators or other aids are permitted. Show all of your work for full credit.

Questions:

1. A 20m tree has has been bent in a storm and makes an angle of 60° with the ground. Some sap is moving down the tree moving at speed 2 m/min. How fast is the distance from the sap to the ground decreasing when the sap is half way down the tree?

Solution: Call the distance from the sap to the ground x and call the distance along the tree from the sap to the ground y. We know that dy/dt is decreasing at a rate of 2 m/min and would like to find dx/dt.



Using the above figure and trigonometry we see that

$$\sin\left(\frac{\pi}{3}\right) = \frac{x}{y}.$$

We know that $\sin \pi/3 = \sqrt{3}/2$ and we rearrange the equation to read

$$\frac{\sqrt{3}}{2} y = x.$$

We now differentiate both sides with respect to t to achieve

$$\frac{\sqrt{3}}{2}\frac{dy}{dt} = \frac{dx}{dt}.$$

With some substitution we find the desired value.

$$\frac{dx}{dt} = \frac{\sqrt{3}}{2}(-2 \text{ m/min}) = -\sqrt{3} \text{ m/min}$$

In this way we now know that dx/dt is decreasing at a rate of $\sqrt{3}$ m/min.

Solution 2: We can also use quotient rule after

$$\sin\left(\frac{\pi}{3}\right) = \frac{x}{y}.$$

Let $x' = dx/dt$ and $y' = dy/dt$ then

$$0 = \frac{x'y - xy'}{y^2}$$

or rather

$$\begin{aligned} x'y &= xy' \\ x' &= \frac{x}{y}y'. \end{aligned}$$

We now note that $x/y = \sin\pi/3$ and so

$$x' = \sin\left(\frac{\pi}{3}\right)y' = \frac{\sqrt{3}}{2}(-2) = -\sqrt{3}.$$

Solution3: We can try to use the Pythagorean Theorem but we have a hard time with it and end up having the use more or less the same trig anyway. Call the remaining side length z so then

$$x^2 + z^2 = y^2.$$

We don't like having z since we know nothing about dz/dt. Let's get rid of it now in favour of y and x. Using some trig we know that $\cos\pi/3 = z/y$ so $z = y\cos\pi/3$. There follows

$$x^2 + \cos^2(\pi/3)y^2 = y^2$$

and

$$\begin{aligned} x^2 &= \left(1 - \cos^2(\pi/3)\right)y^2 \\ &= \sin^2(\pi/3)y^2 \end{aligned}$$

with our favourite trig identity. Taking now the derivative of both sides in t we see

$$2xx' = 2\sin^2(\pi/3)yy'$$

so

$$x' = \sin^2(\pi/3)\frac{y}{x}y'.$$

Recall that $y/x = 1/\sin(\pi/3)$ to finally achieve

$$\begin{aligned} x' &= \frac{\sin^2(\pi/3)}{\sin(\pi/3)}y' \\ x' &= \sin(\pi/3)y' \end{aligned}$$

as before.