Math 190 Homework 3: Due Monday October 5

The assignment is due at the beginning of class on the due date. You are expected to provide full solutions, which are laid out in a linear coherent manner. Your work must be your own and must be self-contained. Your assignment must be stapled with your name and student number at the top of the first page.

Questions:

1. Find all x satisfying

$$\ln(2x - 3) - \ln(5) = 8.$$

2. Find all x satisfying

$$\ln\left((x - 1)^x\right) = 0.$$

3. Find all x satisfying

$$e^{2x} - 4e^x + 4 = 0.$$

4. Many natural phenomena obey power rules. That is

$$Y = CX^m$$

where C and m are constants which depend on the particular application. For example in physics we have the Stephan-Boltzmann equation where Y is the power emitted by a star with temperature X. In forestry we have models of tree size distribution where Y is the number of trees with stem size X. Other examples include frequency of words in most languages, population of cities, and rate of reaction in chemistry.

   (a) Let $y = \ln Y$ and $x = \ln X$. Express y in terms of x assuming that $Y = CX^m$. Note that C and m are fixed constants.

   (b) Suppose we made a plot of y as a function of x. What would the graph look like?

5. In this problem you will prove the identity

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

as seen in class. First let $z_1 = \log_b(x)$ and $z_2 = \log_b(y)$. Rewrite these two equations using exponents instead of logarithms. Use your knowledge of exponent rules to manipulate the equations until you achieve $z_1 + z_2 = \log_b(xy)$. Make sure that you explain each step.

6. **Bonus** Prove the other two logarithm identities.