

Math 190 Homework 8: Due Monday November 16

The assignment is due at the beginning of class on the due date. You are expected to provide full solutions, which are laid out in a linear coherent manner. Your work must be your own and must be self-contained. Your assignment must be stapled with your name and student number at the top of the first page.

Questions:

1. A hollow tree in the shape of a cylinder with radius 40 cm is filling with water during a rain storm. You notice that the height of the water filling the tree is increasing at a rate of 10 cm/min. At what rate is the volume of water contained in the tree increasing when the height of the water is 3 meters.
2. A conical irrigation tank is being filled with water. The top of the tank has a radius of 2m and the tank is 6m high. If water is being pumped in at a rate of $4 \text{ m}^3/\text{s}$ at what rate is the height of the water changing when the height is 3 m. Note that the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the height.

Hint: This problem is more challenging because both r and h are functions of time. Consider using similar triangles to remove one of these variables.

3. Express the area of the half circle as an integral (we will compute this later in lecture).
4. Consider the function

$$f(x) = 2x + 1.$$

- (a) Find the area under the curve $f(x)$ on the interval $[0, 4]$ using the formula for area of a triangle and the formula for area of a rectangle.
 - (b) Approximate this area using Riemann Sums. Use right endpoints and $n = 4$. Is your approximation an overestimate or underestimate? Explain why you expected this.
5. Consider again the function

$$f(x) = 2x + 1.$$

Compute the area under the curve (again on the interval $[0, 4]$) exactly by computing the following limit

$$\lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} f(x_i) \Delta x.$$

You will need to use summation rules as well as the following summation formula

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$