## Math 190 Homework 9: Solutions

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*The assignment is due at the beginning of class on the due date. You are expected to provide full solutions, which are laid out in a linear coherent manner. Your work must be your own and must be self-contained. Your assignment must be stapled with your name and student number at the top of the first page.*

**Questions:**

1. Recall the following integral rules where $k$ is a constant
   - $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
   - $\int_a^b kf(x)dx = k\int_a^b f(x)dx.$

   Note that these rule works for indefinite integrals as well.
   If we know that
   - $\displaystyle\int_{-2}^{2} f(x)dx = 5$
   - $\displaystyle\int_{-2}^{2} g(x)dx = -1$

   then compute

   (a) $\displaystyle\int_{-2}^{2} \big(3f(x) - 2g(x)\big)dx$

   (b) $\displaystyle\int_{-2}^{2} \big(5f(x) + 7g(x)\big)dx$

   (c) $\displaystyle\int_{-2}^{2} 4dx$

   **Solution:**
   (a)

$$\begin{aligned}\int_{-2}^{2} (3f(x) - 2g(x))\,dx &= \int_{-2}^{2} 3f(x)dx - \int_{-2}^{2} 2g(x)dx \\ &= 3\int_{-2}^{2} f(x)dx - 2\int_{-2}^{2} g(x)dx \\ &= 3(5) - 2(-1) \\ &= 17\end{aligned}$$

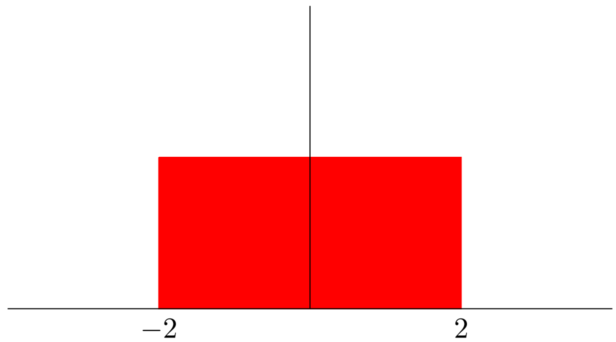
   (b)

$$\begin{aligned}\int_{-2}^{2} (5f(x) + 7g(x))\,dx &= \int_{-2}^{2} 5f(x)dx + \int_{-2}^{2} 7g(x)dx \\ &= 5\int_{-2}^{2} f(x)dx + 7\int_{-2}^{2} g(x)dx \\ &= 5(5) + 7(-1) \\ &= 18\end{aligned}$$

(c) We can compute this integral several ways. We can compute the integral directly:

$$\int_{-2}^{2} 4dx = 4x\Big|_{-2}^{2} = 4(2) - 4(-2) = 16$$

or draw the graph of the function $f(x) = 4$ and use the formula for the area of a rectangle




And so

$$\text{Area} = \text{length} \cdot \text{width} = 4 \cdot 4 = 16.$$

2. Compute the following definite integral

$$\int_{1}^{2} \left(2x^3 - 3\sqrt{x} - \frac{5}{x^2}\right) dx.$$

**Solution:** We compute the anti-derivative of each term:

$$\begin{aligned}\int_{1}^{2} \left(2x^3 - 3\sqrt{x} - \frac{5}{x^2}\right) dx &= \frac{1}{2}x^4 - 2x^{3/2} + 5x^{-1}\Big|_{1}^{2} \\ &= \frac{1}{2}2^4 - 2 \cdot 2^{3/2} + 5 \cdot 2^{-1} - \left(\frac{1}{2}1^4 - 2 \cdot 1^{3/2} + 5 \cdot 1^{-1}\right) \\ &= 7 - 4\sqrt{2}\end{aligned}$$

3. Compute the following definite integral

$$\int_{2}^{3} \left(4e^x - \frac{4}{x}\right) dx.$$

**Solution:** We compute the anti-derivative of each term:

$$\int_{2}^{3} \left(4e^x - \frac{4}{x}\right) dx = 4e^x - 4\ln x\Big|_{2}^{3} = 4e^3 - 4e^2 - 4\ln 3 + 4\ln 2.$$

4. Suppose

$$\int_2^5 f(x)dx = 7.$$

Find

$$\int_5^2 f(x)dx.$$

**Solution:** We solve this problem in two ways. One is to suppose we have the anti-derivative $F(x)$ and consider

$$7 = \int_2^5 f(x)dx = F(5) - F(2).$$

Now

$$\int_5^2 f(x)dx = F(2) - F(5) = -(F(5) - F(2)) = -\int_2^5 f(x)dx = -7.$$

Alternative we can add the two integrals together:

$$\int_2^5 f(x)dx + \int_5^2 f(x)dx = \int_2^2 f(x)dx = 0.$$

Going from 2 to 5 and then from 5 back to 2 yields no net movement. Rearranging the above gives

$$\int_5^2 f(x)dx = -\int_2^5 f(x)dx = -7.$$

5. Compute the following indefinite integral

$$\int \frac{2x + x^3}{\sqrt{x}} dx.$$

**Solution:** To compute this integral we split the sum

$$\begin{aligned}\int \frac{2x + x^3}{\sqrt{x}} dx &= \int \left( \frac{2x}{\sqrt{x}} + \frac{x^3}{\sqrt{x}} \right) dx \\ &= \int \left( 2\sqrt{x} + x^{5/2} \right) dx \\ &= \frac{4}{3}x^{3/2} + \frac{2}{7}x^{7/2} + C.\end{aligned}$$

Note that since this is an indefinite integral we can add any constant $C$.