Work through the following problems while the instructor and TA circulate. When you have completed the problems (to the satisfactory of the facilitators) you can spend the rest of the lab working on the weeks homework.

Warm up: Compute the following integrals using substitution:

•
$$\int e^{-x} dx$$

•
$$\int e^{6x} dx$$

•
$$\int \sin(-3x) dx$$

•
$$\int \frac{e^x}{e^x + 1} dx$$

•
$$\int x^3 \sqrt{x^2 + 1} dx$$

•
$$\int \frac{\cos x}{\sin^2 x} dx$$

Questions:

1. Compute the following definite integrals in two ways

$$\int_{1}^{2} \frac{\left(\ln x\right)^2}{x} dx.$$

First by changing the limits of integration during substitution and then by finding the indefinite integral and then substituting.

2. A function is called *even* if

$$f(-x) = f(x)$$

for all values of x.

(a) Sketch the graph of an even function. Argue for why you think

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$

(b) Split the integral as follows:

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$$

Using the substitution u = -x on the first integral prove that for any even function we have

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$