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Math 190: Midterm Practice Problems

1. a) $\left(e^{x\cos x}\right)' = e^{x\cos x}\left(\cos x - x\sin x\right)$.

chain / product

b) $\left[\dfrac{\sin x}{3(x+1)}\right]' = \dfrac{3(x+1)\cos x - 3\sin x}{\left[3(x+1)\right]^2}$.

quotient.

c) $\left(\sqrt{e^{2x} + \cos(x+x^{1/3})}\right)'$

lots of chain

$$= \frac{1}{2}\left(e^{2x}+\cos(x+x^{1/3})\right)^{-1/2}\left[2e^{2x} - \sin(x+x^{1/3})\cdot\left(1+\tfrac{1}{3}x^{-2/3}\right)\right]$$

$$= \frac{1}{2}\left(e^{2x}+\cos(x+x^{1/3})\right)^{-1/2}\left(2e^{2x} - \left(1+\tfrac{1}{3}x^{-2/3}\right)\sin(x+x^{1/3})\right).$$

d) $\left[\ln\left(\dfrac{1}{x\sqrt{3x+1}}\right)\right]'$

$$= \frac{1}{\frac{1}{x\sqrt{3x+1}}}\left(\left(x\sqrt{3x+1}\right)^{-1}\right)'$$

all the rules.

$$= x\sqrt{3x+1}\left(-\left(x\sqrt{3x+1}\right)^{-2}\left(\sqrt{3x+1} + x\cdot\tfrac{1}{2}(3x+1)^{-1/2}\cdot 3\right)\right)$$

OR

$$\ln\left(\frac{1}{x\sqrt{3x+1}}\right) = \ln(1) - \ln\left(x\sqrt{3x+1}\right)$$

← log rules.

$$= 0 - \ln x - \ln\left((3x+1)^{1/2}\right)$$

$$= -\ln x - \frac{1}{2}\ln(3x+1).$$

So, $\dfrac{d}{dx}\left[\ln\left(\dfrac{1}{x\sqrt{3x+1}}\right)\right] = \dfrac{-1}{x} - \dfrac{1}{2}\dfrac{1}{3x+1}\cdot 3$

3

2. First find the derivative (slope)

$$f'(x) = 3x^2 - x - 2$$

To find where the slope is zero set

$$f'(x) = 0$$

So, solve,

$$3x^2 - x - 2 = 0$$

$$3x^2 - 3x + 2x - 2 = 0 \quad \text{(factor)}$$

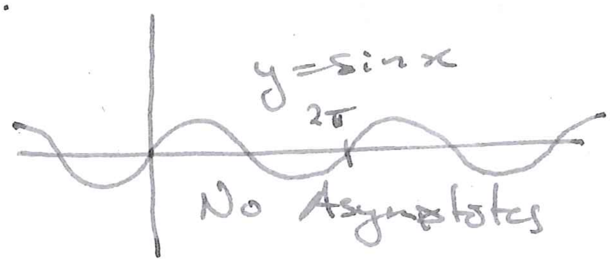
$$3x(x-1) + 2(x-1) = 0$$

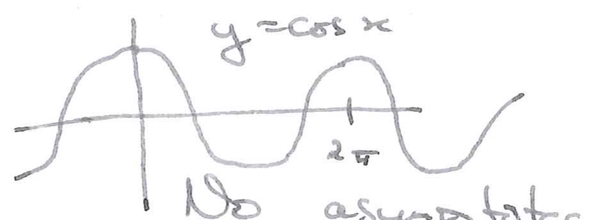
$$(3x+2)(x-1) = 0$$

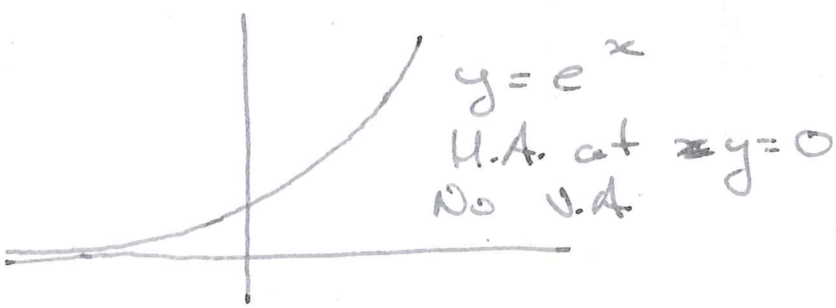
$$\Rightarrow \quad x = 1, -2/3.$$

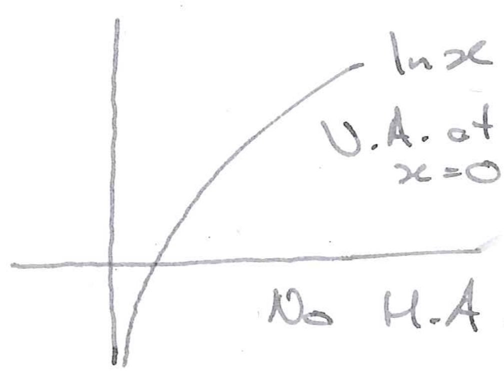
So, horizontal tangent lines at $x=1$ and $x = -2/3$.

3.









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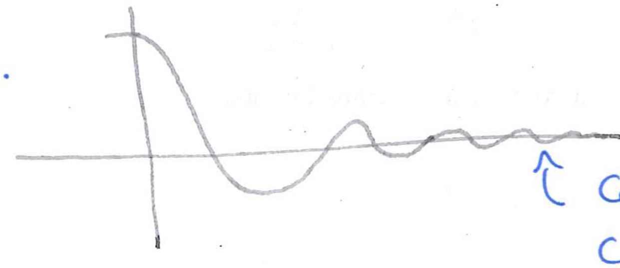
4. A function can cross its horizontal asymptote infinitely many times.

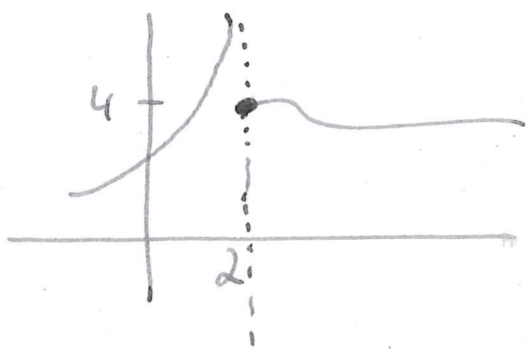
Observe the following function:

$\lim_{x \to \infty} f(x) = 0.$



continues to cross but approaches zero.

5. Yes, observe the following function:



We see $\lim_{x \to 2^-} f(x) = \infty$ and $f(2) = 4$.

6. Horizontal asymptotes. Consider the following limits

- $$\lim_{x \to \infty} \frac{x-2}{x^2+2x-3} = \lim_{x \to \infty} \frac{x/x^2 - 2/x^2}{x^2/x^2 + 2x/x^2 - 3/x^2}$$
$$= \lim_{x \to \infty} \frac{\overset{\to 0}{1/x^2} - \overset{\to 0}{2/x^2}}{1 + \underset{\to 0}{2/x} - \underset{\to 0}{3/x^2}} = \frac{0-0}{1+0-0} = \underline{\underline{0}}.$$

- $$\lim_{x \to -\infty} \frac{x-2}{x^2+2x-3} = \lim_{x \to -\infty} \frac{1/x^2 - 2/x^2}{1 + 2/x - 3/x^2} = \frac{0-0}{1+0-0} = \underline{\underline{0}}$$

$\Rightarrow$ Horizontal asymptote at $y = 0$.

8

6. Vertical Asymptotes.

Identify candidates. $\dfrac{x-2}{x^2+2x-3} = \dfrac{x-2}{(x-1)(x+3)}$.

Try $x=1$ and $x=-3$.

Consider the four relevant one sided limits

- $\lim\limits_{x\to 1^+} \dfrac{x-2}{(x-1)(x+3)} = -\infty \qquad \left(\dfrac{-}{(+)(+)}\right)$

- $\lim\limits_{x\to 1^-} \dfrac{x-2}{(x-1)(x+3)} = +\infty \qquad \left(\dfrac{-}{(-)(+)}\right)$

$\Rightarrow$ V.A. at $x=1$.

- $\lim\limits_{x\to -3^+} \dfrac{x-2}{(x-1)(x+3)} = +\infty \qquad \left(\dfrac{-}{(-)(+)}\right)$

- $\lim\limits_{x\to -3^-} \dfrac{x-2}{(x-1)(x+3)} = -\infty \qquad \left(\dfrac{-}{(-)(-)}\right)$

$\Rightarrow$ V.A. at $x=-3$.

7. a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)+3}-\sqrt{2x+3}}{h} \cdot \frac{\sqrt{2(x+h)+3}+\sqrt{2x+3}}{\sqrt{2(x+h)+3}+\sqrt{2x+3}}$$

$$= \lim_{h\to 0} \frac{2(x+h)+3-(2x+3)}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}$$

$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)+3}+\sqrt{2x+3}} = \frac{2}{\sqrt{2x+3}+\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}.$$

(5) 7. b) Using Chain Rule

$$f(x) = (2x+3)^{1/2}$$

$$f'(x) = \frac{1}{2}(2x+3)^{-1/2} \cdot 2$$

$$= \frac{1}{\sqrt{2x+3}}$$

8. a) Consider the two one sided limits noting that $|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$

- $\lim_{x \to 1^+} \frac{|x-1|}{x^2+2x-3} = \lim_{x \to 1^+} \frac{(x-1)}{(x-1)(x+3)} = \lim_{x \to 1^+} \frac{1}{x+3} = \frac{1}{4}.$
- $\lim_{x \to 1^-} \frac{|x-1|}{(x^2+2x-3)} = \lim_{x \to 1^-} \frac{-(x-1)}{(x-1)(x+3)} = \lim_{x \to 1^-} \frac{-1}{x+3} = \frac{-1}{4}.$

$\Rightarrow$ Since the two one sided limits are different the full limit

$\lim_{x \to 1} \frac{|x-1|}{x^2+2x-3}$ does not exist.

b) $$\lim_{x \to 10} \frac{\sqrt{x-1}-3}{x-10} \cdot \frac{\sqrt{x-1}+3}{\sqrt{x-1}+3}$$

$$= \lim_{x \to 10} \frac{x-1-9}{(x-10)(\sqrt{x-1}+3)}$$

$$= \lim_{x \to 10} \frac{x-10}{(x-10)(\sqrt{x-1}+3)}$$

$$= \lim_{x \to 10} \frac{1}{\sqrt{x-1}+3} = \frac{1}{\sqrt{10-1}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

5

9. Find all $x \in [0, \pi)$ such that the function

$$g(x) = \cos(2x) - 2x$$

has horizontal tangent lines.

Compute,

$$g'(x) = -2\sin(2x) - 2$$

and set $g'(x) = 0$. So,

$$-2\sin(2x) - 2 = 0.$$

$$\sin(2x) = -1$$

So $2x = \frac{3\pi}{2}$

$$x = \frac{3\pi}{4} \in [0, \pi).$$

10. a) $f'(x) = 5x^4$

b) Power Rule:

c) Product Rule: $f(x) = x^3 \cdot x^2$

$$f'(x) = 3x^2 \cdot x^2 + x^3 \cdot 2x$$

$$= 3x^4 + 2x^4$$

$$= 5x^4.$$

Quotient Rule:

$$f(x) = \frac{x^6}{x}$$

$$f'(x) = \frac{6x^5 \cdot x - x^6}{x^2}$$

$$= \frac{5x^6}{x^2}$$

$$= 5x^4$$

Chain Rule: $f(x) = (x^2)^{5/2}$.

$$f'(x) = \frac{5}{2}(x^2)^{3/2} \cdot 2x$$

$$= 5x^3 \cdot x$$

$$= 5x^4.$$