Dec. 2

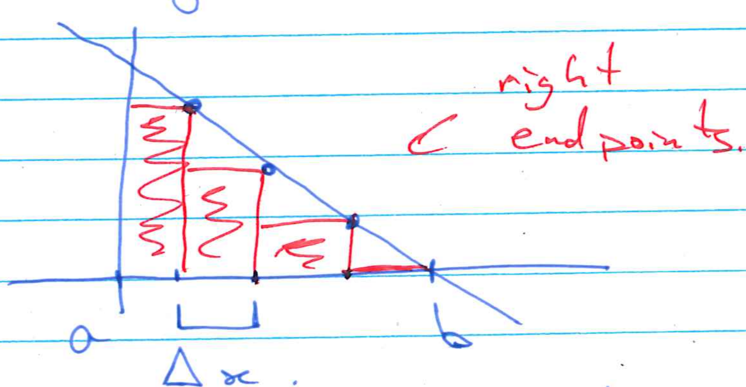
1

- HW11 (not for marks)
  Solutions Friday.
- Exam Practice Problems: Friday.
- all materials returned Friday.
- grades posted Friday.
- Course Evals
  - do them.

Exam: Dec. 14 at 12noon.
(2.5 hours).

Riemann Sums: What do I have to know?
(see learning goals).

Given a simple function (polynomial) approximate the area under the curve using Riemann Sums.
(specify either right / left endpoints)
given a number of bars, n.



$n = \dfrac{b-a}{\Delta x}$

$\Delta x = \dfrac{b-a}{n}$.

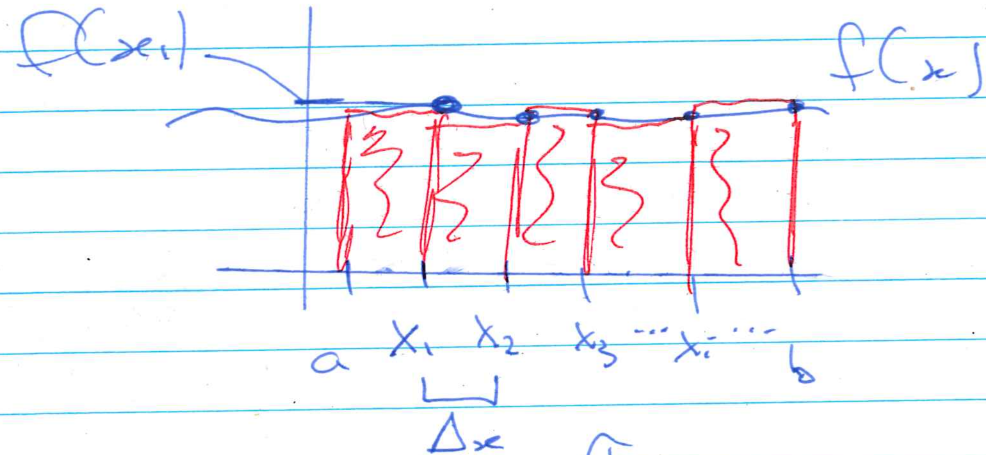
Explain why $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

is the definition of the integral.
(picture).

2

Note: Not required to compute this limit ($n \to \infty$).
This requires a summation formula.

What is Δx?
What are the x_i? $f(x_i)$?



n – number of bars.

$$\Delta x = \frac{b-a}{n}.$$

these are the x_i.

(see practice problems).

An Integration "word" problem:

A strange alien tree is currently 1 meter tall. The rate of which it grows is given by

growth rate / rate of change of height. $\rightarrow$ $r(t) = \dfrac{4t}{t^2+1}$ in m/week.

How tall is the tree after 10 weeks.

$$h(t) = \int \frac{4t}{t^2+1}\,dt.$$

Let $u = t^2+1$.
$\frac{du}{dt} = 2t \Rightarrow \frac{du}{2} = t\,dt$.

$$\int \frac{4t}{t^2+1}\,dt = \int \frac{2}{u}\,du = 2\int \frac{1}{u}\,du$$
$$= 2\ln u + C$$

$h(t) = 2\ln(t^2+1) + C$.

when $t=0$: $h(0) = 1$.

$$1 = h(0) = 2\ln(0^2+1) + C$$
$$= 2\ln(1) + C$$
$$= 0 + C \Rightarrow C = 1.$$

$h(t) = 2\ln(t^2+1) + 1$.

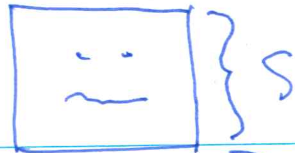
$h(10) = 2\ln(101) + 1 \approx 10.2\,m$.

Related Rates:

Steps:
1. Picture with notation
2. Given / Required Rate.
3. Equation.
4. Chain Rule
5. Solve / Substitute.

Example: A particular alien "Square-fish" grows in the shape of a square. If the side lenght is increasing at a rate of 3 mm/day. How fast is the area changing when side length is 40mm.

(a)

1. 

Area - A.

2. $\frac{ds}{dt} = 3$ (Have)

Want: $\frac{dA}{dt}$.

s is like the inside function

3. Equation.

$$A = s^2.$$

4. Chain Rule:

$$\frac{dA}{dt} = \frac{d}{dt}(s^2)$$

$$= 2s \cdot \frac{ds}{dt}.$$

5. Sub/solve:

$$\frac{dA}{dt} = 2(40) \cdot 3$$

$$= 240 \text{ mm}^2/\text{day}.$$

Example: The rate of my hunger (hunger units / hour) is given by

$$r(t) = \frac{1}{(2-t)^2}.$$

I am currently at 2 hunger units. How hungry am I in one hour?

rumbble

hunger

$$h(t) = \int \frac{1}{(2-t)^2}\, dt.$$

let $u = 2 - t$.

$du = -dt$.

(5)

$$\int \frac{1}{(2-t)^2}\,dt = -\int \frac{1}{u^2}\,du.$$

$$= -\int u^{-2}\,du.$$

$$= -(-u^{-1}) + C.$$

$$= \frac{1}{u} + C.$$

$$h(t) = \frac{1}{2-t} + C.$$

Solve for C using $h(0) = 2$.

$$h(0) = 2 = \frac{1}{2-0} + C$$

$$2 = \frac{1}{2} + C$$

$$C = 3/2.$$

$$h(t) = \frac{1}{2-t} + 3/2.$$

$$h(1) = \frac{1}{2-1} + 3/2 = 1 + 3/2$$

$$= 5/2 \text{ units.}$$