1 Nov. 16

- HW8 Due.
- Lab 9/HW 9 to be posted.

Indefinite Integrals:

Clicker Q: Is $\frac{1}{3}x^3$ the only anti-derivative of x^2?

39% A) Yes
57% B) No. ⟸

What is another one?

$\frac{1}{3}x^3 + 2$ ← still anti-derivative

$\frac{1}{3}x^3 - 1$

derivative will kill constant.

The general anti-derivative of $f(x) = x^2$ is

$$F(x) = \frac{1}{3}x^3 + C$$

for any constant C.

②

We write,

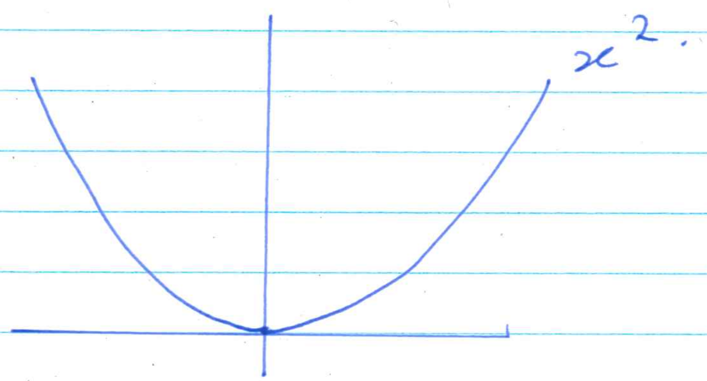
- $\int x^2\,dx = \frac{1}{3}\cancel{x}x^3 + C.$

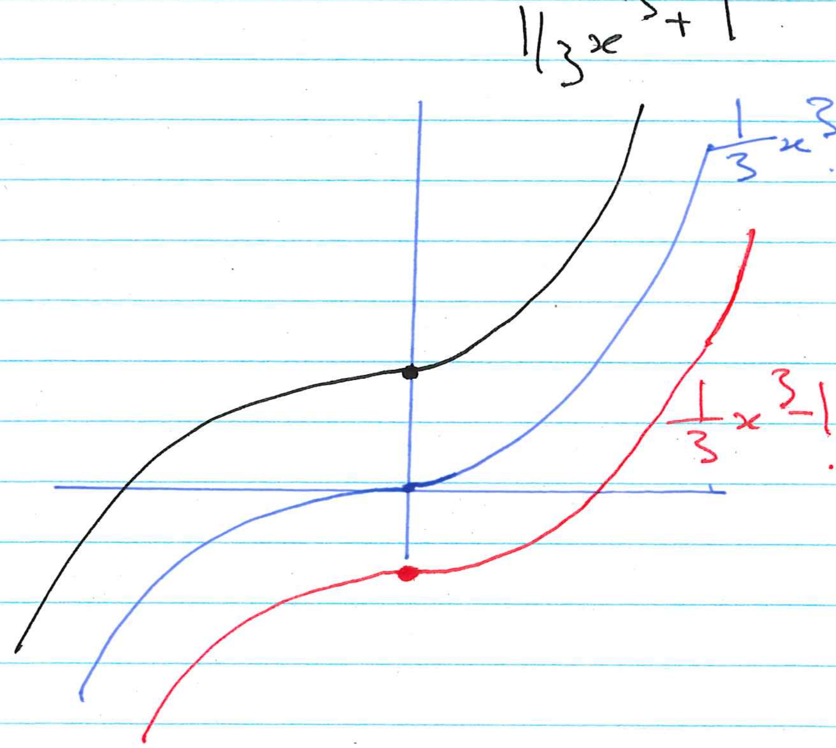
indefinite integral.

function.
(family of functions)

- $\int_1^3 x^2\,dx \leftarrow$ number.

definite





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Does this constant mess up our definite integrals?

$$\int_1^3 x^2\,dx = F(b) - F(a)$$

$b = 3$, $a = 1$.

where $F(x)$ is any anti-derivative.

We could use $F(x) = \frac{1}{3}x^3 + 2$.

$F(3) - F(1)$

$$= \frac{1}{3}(3)^3 + \cancel{2} - \frac{1}{3}(1)^3 - \cancel{2}$$

$\Rightarrow$ constants will cancel out.

Example: Find $\int e^{2x}\,dx$.

ClickerQ: A) $e^{2x} + C$

B) $2e^{2x} + C$

$\rightarrow$ C) $\frac{1}{2}e^{2x} + C$

(4)

$$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C.$$

$$\left(\text{Check: } \left(\frac{1}{2}e^{2x} + C\right)' \qquad \frac{1}{2}e^{2x}\cdot 2 + 0 = e^{2x} \checkmark \right)$$

$$F(x) = \frac{1}{2}e^{2x} + C.$$

Find a function $F(x)$ so that $F'(x) = e^{2x}$ and $F(0) = 2$.

We want $F(0) = 2 = \frac{1}{2}e^{2\cdot 0} + C.$

find C to satisfy our condition.

$$2 = \frac{1}{2} + C$$

$$\Rightarrow \quad C = 3/2.$$

$$\Rightarrow \quad F(x) = \frac{1}{2}e^{2x} + 3/2.$$

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Example: Find a function $F(x)$ with $F'(x) = 3\sin(2x)$ and $F(0) = -2$.

First find general anti-derivative.

$$F(x) = -\frac{3}{2}\cos(2x) + C.$$

Check: $\left[-\frac{3}{2}\cos(2x)\right]'$

$$= 3\sin(2x).$$

(aside: $F'(x) = 3\sin(2x)$.

(Easier: $F'(x) = 3\sin(x)$.
$F(x) = -3\cos(x)$).

$F'(x) = 3\sin(2x)$.
$F(x) = \text{S}\cos(2x)$)

something.

Something $= -3/2$ by chain rule.

⑥

find.

$$\left(A\cos(2x) \right)' = 3\sin(2x).$$

$$A(-\sin(2x)\cdot 2) = 3\sin(2x).$$

$$-2A\,\cancel{\sin(2x)} = 3\,\cancel{\sin(2x)}.$$

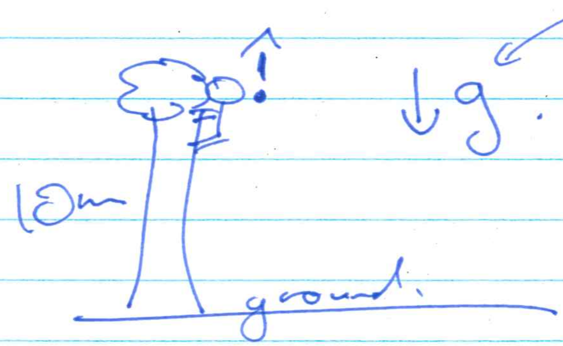
$$A = -\frac{3}{2}.$$

$$F(x) = \frac{1}{2}e^{2x} + 3/2.$$

Come back to this on Wednesday.

Now with a little integration we can do all the high school physics problems.

Example: acceleration due to gravity is constant. You throw the object up with velocity 2m/s.



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Let's find an equation for $S(t)$ the distance from the ground.

$$\boxed{\int 2\,dx = 2x + C}$$

acceleration → $a(t) = -g$.

$$v(t) = \int a(t)\,dt.$$

$$v(t) = \int -g\,dt.$$

$$= -gt + C.$$

velocity.

take the derivative of $v(t)$.

$$v'(t) = (-gt + C)'$$

$$= -g.$$

$$a(t) = -g = -9.8$$

$$v(t) = -9.8t + C.$$

How to find C?

We know $v(0) = 2$.

$$2 = v(0) = -9.8 \cdot 0 + C.$$

$$C = 2.$$

(8).

velocity

$$v(t) = -9.8t + 2.$$

position → $s(t) = \int v(t)\,dt.$ integration

$$s(t) = \frac{-9.8}{2}t^2 + 2t + C.$$

$$\left(\begin{array}{l} f(x) = x \\ F(x) = \frac{1}{2}x^2. \end{array} \right)$$

$$s(0) = 10. = \frac{-9.8}{2}0^2 + 2\cdot 0 + C$$

$$C = 10.$$

$$s(t) = \frac{-9.8}{2}t^2 + 2t + 10.$$