Nov. 18

- HW9 # 2 (negative changed).

$$\int_{-1}^{2} \longrightarrow \int_{1}^{2}$$

- Quiz #5 Friday 27th.
  - Last week / This week
  - materials to be posted.

Indefinite Integrals.

1) $\int 3x^4\,dx$

2) $\int \frac{2}{\sqrt{x}}\,dx$

3) $\int e^x\,dx$

4) $\int (7\sin x + 8\cos x)\,dx$

5) $\int \frac{1}{x}\,dx$.

Clickers:

A) 0 done
B) 1 done
C) 2
D) 3
E) 4.

1) $\int 3x^4\,dx$

$= \frac{3}{5}x^5 + C.$

(General. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$).

2) $\int \frac{2}{\sqrt{x}}\,dx = \int 2x^{-1/2}\,dx.$

$= 2\,\frac{x^{1/2}}{1/2} + C.$

$= 2 \cdot \frac{2}{1}\,x^{1/2} + C$

$= 4x^{1/2} + C.$

3) $\int e^x\,dx = e^x + C.$

4) $\int (7\sin x + 8\cos x)\,dx$.

$= \int 7\sin x\,dx + \int 8\cos x\,dx$.

$= -7\cos x + 8\sin x + C$.

5) $\int \frac{1}{x}\,dx = \ln x + C$.

For more challenging integrals we will need new techniques.

Substitution Method.

Examples: $\int (2x+1)^2\,dx$.

Rather than guessing let's make a substitution.

Let us call $u = 2x+1$.

$\int (2x+1)^2\,dx = \int u^2\,dx$ ← get rid of dx.

↑ change all x's to u's.

(4)

$u = 2x + 1$

$\frac{du}{dx} = 2 \Rightarrow du = 2dx$.

$\Rightarrow dx = \frac{1}{2} du$.

So,

$$\int (2x+1)^2 dx = \int u^2 \frac{1}{2} du$$

$$= \frac{1}{2} \int u^2 du.$$

Change back to original variable.

$$= \frac{1}{2} \frac{u^3}{3} + C.$$

$$= \frac{1}{6} (2x+1)^3 + C.$$

$$\Rightarrow \int (2x+1)^2 dx = \frac{(2x+1)^3}{6} + C.$$

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Substitution is kind of like anti-chain rule.

Example:

$$\int e^{4x}\,dx$$

Let $u = 4x$.

$$\frac{du}{dx} = 4.$$

$$du = 4\,dx$$

$$dx = \frac{1}{4}\,du.$$

$$\int e^{4x}\,dx = \int e^{u}\,\frac{1}{4}\,du.$$

$$= \frac{1}{4}\int e^{u}\,du.$$

$$= \frac{1}{4}\,e^{u} + C.$$

$$= \frac{1}{4}\,e^{4x} + C.$$

Example:

$$\int x \sin(x^2)\, dx .$$

let $u = x$ A

$u = \sin(x)$ B

$u = \sin(x^2)$ C 32%.

$u = x\sin(x^2)$ D

$\rightarrow u = x^2$. E 51%

Why doesn't C work.

$$u = \sin(x^2)$$

$$\frac{du}{dx} = \cos(x^2) \cdot 2x$$

$$du = \cos(x^2) \cdot 2x \, dx$$

$$dx = \frac{du}{\cos(x^2) \cdot 2x}$$

$$\int x \, u \, \frac{du}{\cos(x^2) \cdot 2x}$$

(7)

let $u = x^2$

$\frac{du}{dx} = 2x$.

$du = 2x\,dx$.

$\frac{1}{2}du = x\,dx$.

$$\int \underbrace{\sin(x^2)}_{\sin(u)} \underbrace{x\,dx}_{\frac{1}{2}du} = \frac{1}{2}\int \sin u\,du.$$

$$= \frac{-1\cos u}{2} + C.$$

$$= -\frac{1}{2}\cos(x^2) + C.$$

Example: $\int \frac{x}{\sqrt{2x^2+3}}\,dx$.

let? $u = x^2$, $u = 2x^2+3$, $u = \sqrt{2x^2+3}$ } one of these works.