Nov. 20

- Quiz #4 returned.
  - no names.
- Practice Problems / Learning Objectives.
- Quiz #5 Fri. 27th.
  - this week / last week.

Last Class: $\int \frac{x}{\sqrt{2x^2+3}}\,dx$.

Clicker Q:

A) $u = x^2$

→ B) $u = 2x^2+3$

C) $u = \sqrt{2x^2+3}$.

Let $u = 2x^2+3$

$\frac{du}{dx} = 4x \Rightarrow dx = \frac{1}{4x}du.$

$x\,dx = \frac{1}{4}du.$

$\int \frac{x\,dx}{\sqrt{2x^2+3}}$ $\quad \frac{1}{4}du$

$= \int \frac{1}{\sqrt{u}} \frac{1}{4} du.$

Alternatively, $dx = \frac{1}{4x} du.$

$\int \frac{x\,dx}{\sqrt{2x^2+3}} = \int \frac{\cancel{x}}{\sqrt{u}} \frac{1}{4\cancel{x}} du$

$= \frac{1}{4} \int \frac{du}{\sqrt{u}}.$

(2)

$$\frac{1}{4}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{4}\int u^{-1/2}\,du.$$

$$= \frac{1}{4}\left(\frac{2}{\cancel{3}}\sqrt{u}\right) + C.$$

$$\left(\int u^{-1/2}\,du = \frac{u^{1/2}}{1/2}.\right)$$

$$= \frac{1}{4}\cdot 2\sqrt{u} + C.$$

$$= \frac{1}{2}\sqrt{u} + C$$

$$= \frac{1}{2}(2x^2+3)^{1/2} + C.$$

③

Check with Chain Rule.

$$\frac{d}{dx}\left[\frac{1}{2}(2x^2+3)^{1/2}+C\right]$$

$$=\frac{1}{4}(2x^2+3)^{-1/2}\cdot 4x.$$

$$=\frac{x}{\sqrt{2x^2+3}}$$ //.

Example: Find $\int x\sqrt{x+3}\,dx$.

let $u = x+3 \Rightarrow x = u-3.$

$\frac{du}{dx} = 1 \Rightarrow du = dx.$

$$= \int x\sqrt{u}\,du = \int (u-3)\sqrt{u}\,du.$$

$$= \int u\sqrt{u}\,du + \int -3\sqrt{u}\,du.$$

$$= \int u^{3/2}\,du - 3\int \sqrt{u}\,du.$$

(a)

$$= \frac{2}{5} u^{5/2} - \cancel{3}\frac{2}{\cancel{3}} u^{3/2} + C.$$

$$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C.$$

Definite Integrals with Substitution

$$\int_{\pi/2}^{\pi} \sin^2 x \cos x \, dx.$$

There are two ways we can go.

One way: find indefinite integral first, and substitute at the end.

$$\int \sin^2 x \cos x \, dx.$$

let $u = \sin x$, ~~$u = \cos x$~~

$$\frac{du}{dx} = \cos x$$

$$du = \cos x \, dx.$$

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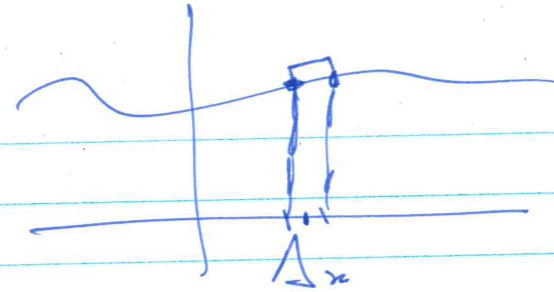
$$\int \overbrace{\sin^2 x}^{u^2} \cdot \overbrace{\cos x \cdot dx}^{du}.$$



$$= \int u^2 \, du$$

$$= \frac{1}{3} u^3 + C.$$

$$= \frac{1}{3} \sin^3 x + C.$$

To find: $$\int_{\pi/2}^{\pi} \sin^2 x \cos x \, dx$$

$$= F(\pi) - F(\pi/2).$$

$$F(x) = \frac{1}{3} \sin^3 x.$$

$$= \frac{1}{3} \{\underset{\overset{\parallel}{0}}{\sin(\pi)}\}^3 - \frac{1}{3} \{\underset{\overset{\parallel}{1}}{\sin(\pi/2)}\}^3.$$

$$= -\frac{1}{3} \cdot 1^3 = -1/3$$

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OR: Change the limits of integration

$$\int_{\pi/2}^{\pi} \sin^2 x \cos x \, dx$$

let ~~u = sin~~

$u = \sin x$

$du = \cos x \, dx.$

When $x = \pi/2$: $u = \sin(\pi/2) = 1.$

$u = 1$

When $x = \pi$: $u = \sin(\pi) = 0$

$u = 0.$

$$= \int_{1}^{0} u^2 \, du = \left. \frac{u^3}{3} \right|_{u=1}^{u=0}$$

$$= \frac{0^3}{3} - \frac{1^3}{3} = -1/3$$

Example: $$\int_1^2 \frac{\ln(x)}{x}\,dx.$$

$$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x}.$$

$$du = \frac{1}{x}\,dx.$$

- $$\int_{x=1}^{x=2} \ln x \overbrace{\frac{1}{x}\,dx}^{du} = \int_{u=0}^{u=\ln(2)} u\,du.$$

when: ~~ued~~ $x = 1 \Rightarrow u = 0$
$x = 2 \Rightarrow u = \ln(2)$.

$$= \left.\frac{u^2}{2}\right|_{u=0}^{u=\ln(2)}$$

$$= \frac{(\ln 2)^2}{2} \;//.$$

- $$\int_{\cancel{1}}^{\cancel{2}} \ln x \frac{1}{x}\,dx = \int u\,du$$

$$= \cancel{\frac{u^3}{3}} \cdot \frac{u^2}{2}.$$

$$= \frac{(\ln x)^2}{2} = F(x).$$

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$$\int \ln x \frac{1}{x}\, dx = \frac{(\ln x)^2}{2} + C.$$

$$\int_1^2 \ln x \frac{1}{x}\, dx = \frac{(\ln x)^2}{2}\Bigg|_{x=1}^{2}$$

$$= \frac{\{\ln(2)\}^2}{2} - \frac{\{\ln(1)\}^2}{2}$$

$$= \frac{\{\ln(2)\}^2}{2}$$