1 Nov. 23

- HW 9 Due
- Quiz #5 Friday.
- HW 10 to be posted.

Example: $\int x \cdot e^x \, dx$.

Integration By Parts (IBP).

- $\int u \, dv = uv - \int v \, du$.

$$\int \underbrace{x}_{u} \underbrace{e^x \, dx}_{dv}.$$

let $u = x$ $\qquad dv = e^x dx$.

$\frac{du}{dx} = 1$ $\qquad \int 1 dv = \int e^x dx$

$du = dx$ $\qquad v = e^x$

$$\int x e^x dx = \int u \, dv$$

$$= uv - \int v \, du.$$

$$= x e^x - \int e^x dx.$$

②

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$
$$= xe^x - e^x + C.$$

$$\left(\begin{aligned}&\underline{\text{Check}}: \quad (xe^x - e^x + C)' \\ &= 1\cdot e^x + xe^x - e^x. \\ &= xe^x \;/\!/.\end{aligned}\right)$$

Integration by parts is like anti-product rule.

We take $u = x$ since differentiating this makes it simpler.

Note we also had to integrate dv.

<u>Example</u>: $\displaystyle\int \overset{u}{x}\, \overbrace{\cos x\, dx}^{dv}.$

let $u = x$ $\qquad dv = \cos x\,dx.$

$du = dx$ $\qquad v = \sin x$

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$$\int u\,dv = uv - \int v\,du.$$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$
$$= x\sin x + \cos x + C.$$

Example: $\int x\ln x\,dx.$

ClickerQ:

64% A) $u = x$ $\quad dv = \ln x\,dx$

32% B) $u = \ln x$ $\quad dv = x\,dx.$

Try A) $u = x$ $\quad dv = \ln x\,dx$
$du = dx$ $\quad v = \int \ln x\,dx.$

differentiating x is easy ... but ... integrating $\ln x$ is hard.

(4)

$$\int x \ln x \, dx.$$

differentiating $\ln x$ is ok.

integrating x is ok.

$u = \ln x \qquad dv = x\,dx.$

$\frac{du}{dx} = 1/x \qquad v = \int x\,dx$

$du = \frac{1}{x}dx \qquad = \frac{1}{2}x^2.$

$$\int u\,dv = uv - \int v\,du.$$

$$\int x \ln x \, dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x^2 \frac{1}{x}\,dx.$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx.$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{1}{2}x^2\right) + C.$$

$$= \frac{x^2 \ln x}{2} - \frac{1}{4}x^2 + C.$$

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Definite integrals with IBP works the same way.

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$$

<u>Example</u>: $\displaystyle\int_0^{\pi/2} x^2 \sin x\,dx.$

$u = x^2 \qquad dv = \sin x\,dx.$

$\frac{du}{dx} = 2x \qquad v = -\cos x$

$du = 2x\,dx$

$$\int_0^{\pi/2} x^2 \sin x\,dx = -x^2\cos x\Big|_0^{\pi/2} - \int_0^{\pi/2}(-\cos x)2x\,dx.$$

$$= -x^2\cos x\Big|_0^{\pi/2} + 2\int_0^{\pi/2} x\cos x\,dx.$$

$$= -\left(\frac{\pi}{2}\right)^2\underbrace{\cos(\pi/2)}_{=0} - \left(-0^2\underbrace{\cos(0)}_{=0}\right) + 2\int_0^{\pi/2} x\cos x\,dx.$$

6).

$$= 2\int_0^{\pi/2} x\cos x\,dx.$$

$u = x$ $\qquad dv = \cos x\,dx$

$du = dx$ $\qquad v = \sin x$

$$= 2\left[x\sin x\Big|_0^{\pi/2} - \int_0^{\pi/2}\sin x\,dx\right]$$

$$= 2\left[\frac{\pi}{2}\overbrace{\sin\left(\frac{\pi}{2}\right)}^{1} - \underbrace{(0)\sin(0)}_{0} - \int_0^{\pi/2}\sin x\,dx\right].$$

$$= \pi + 2\cos x\Big|_0^{\pi/2}$$

$$= \pi + 2\overbrace{\cos(\pi/2)}^{0} - 2\cos(0).$$

$$= \pi - 2.$$