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Nov. 25.

- HW 8 Returned : See Solutions.
- Quiz #5 Friday
- Course Evals.
  - do them.

Warm up!

$$\int \underbrace{\ln x}_{u} \, \underbrace{dx}_{dv}$$

Using integration by Parts

derivative of $u \rightarrow$ $u = \ln x$ $\qquad$ $dv = dx$ $\leftarrow$ anti-derivative of v.

$du = \frac{1}{x}\,dx$ $\qquad$ $v = x$.

$$\left(\frac{du}{dx} = \frac{1}{x}.\right) \qquad \left(\int 1\,dv = \int 1\,dx \quad v = x.\right)$$

$$\int u\,dv = uv - \int v\,du.$$

$$\int \ln x\,dx = x\ln x - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx.$$

$$= x\ln x - \int 1\,dx$$

$$= x\ln x - x + C.$$

Check:

$$\left(x\ln x - x + C \right)'$$

$$= 1 \cdot \ln x + x \frac{1}{x} - 1$$

$$= \ln x + 1 - 1$$

$$= \ln x .$$

Example: $\int e^x \sin x \, dx$.

let $u = e^x$ $\qquad dv = \sin x \, dx$.

$\frac{du}{dx} = e^x$ $\qquad v = \int \sin x \, dx$

$du = e^x \, dx$ $\qquad = -\cos x$

$$\int u \, dv = uv - \int v \, du$$

$$\int e^x \sin x \, dx = -e^x \cos x - \int (-\cos x) e^x dx$$

$$= -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x \, dx}_{dv} .$$

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Did that help? Not really.
Try again? Yes.

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x\,dx \qquad v = \sin x$

$$\int e^x \sin x\,dx = -e^x \cos x + \underbrace{\int e^x \cos x\,dx}.$$

$$\int u\,dv = uv - \int v\,du.$$

$$= e^x \sin x - \int \sin x\, e^x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx + \int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x.$$

$$\int e^x \sin x\,dx = \frac{-e^x \cos x + e^x \sin x}{2} + C.$$

Example: We can combine rules.

$$\int x^3 e^{x^2}\,dx$$

Substitution: $u = x^2$.

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x\,dx.$$

$$\int x^2 e^{x^2} \overbrace{x\,dx}^{\frac{du}{2}}. \quad = \underbrace{\frac{1}{2}\int \overbrace{u}^{w}\, \overbrace{e^u\,du}^{dv}}_{\text{FBP}}.$$

$$\int w\,dv = wv - \int v\,dw.$$

$w = u$

$\frac{dw}{du} = 1$

$dw = du$

$dv = e^u\,du.$

$v = \int e^u\,du$

$v = e^u.$

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$$\frac{1}{2}\int ue^u\,du$$

$$= \frac{1}{2}\left(ue^u - \int e^u\,du\right)$$

$$= \frac{1}{2}ue^u - \frac{1}{2}\int e^u\,du.$$

$$= \frac{1}{2}ue^u - \frac{1}{2}e^u + C.$$

Switch back. $u = x^2$.

$$= \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C.$$

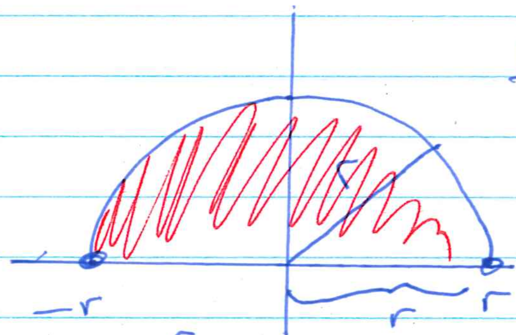
$$\left(= \frac{1}{2}e^{x^2}(x^2-1) + C.\right)$$

(3)

Example: The area of a circle.

Eq. $x^2 + y^2 = r^2$. $\Rightarrow$ $y^2 = r^2 - x^2$

$f(x) = y = \sqrt{r^2 - x^2}$.



Find $\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$. r - constant.

First. 1) $u = \frac{1}{r} x \Rightarrow x = ru$.

$\frac{du}{dx} = \frac{1}{r} \Rightarrow r\,du = dx$.

$$\int_{-1}^{1} \sqrt{r^2 - r^2 u^2}\; r\,du.$$

$x = -r \Rightarrow u = -1$
$x = r \Rightarrow u = 1$

$$= \int_{-1}^{1} \sqrt{r^2(1-u^2)}\; r\,du.$$

$$= \int_{-1}^{1} r\sqrt{1-u^2}\; r\,du. = r^2 \int_{-1}^{1} \sqrt{1-u^2}\,du.$$

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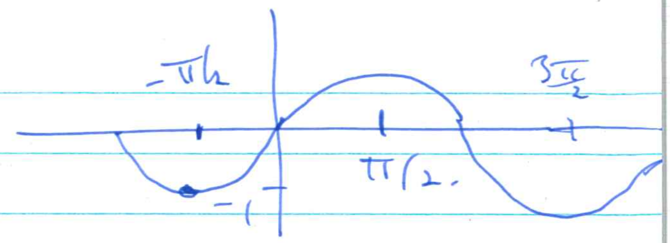
$$= r^2 \int_{-1}^{1} \sqrt{1-u^2}\, du.$$



2) $u = \sin\theta$.

$\frac{du}{d\theta} = \cos\theta$.

$u = -1$
$\theta = -\pi/2$.

$u = 1 \quad \theta = \pi/2$.

$du = \cos\theta \, d\theta$.

$$= r^2 \int_{-\pi/2}^{\pi/2} \sqrt{1 - \underbrace{\sin^2\theta}_{u^2}}\ \overbrace{\cos\theta \, d\theta}^{du}.$$

trig id: $1 - \sin^2\theta = \cos^2\theta$.
$(\sin^2\theta + \cos^2\theta = 1.)$.

$$= r^2 \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2\theta}\ \cos\theta \, d\theta.$$

$$= r^2 \int_{-\pi/2}^{\pi/2} \cos\theta \cdot \cos\theta \, d\theta$$

$$= r^2 \int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta.$$

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$$= r^2 \int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta .$$

3) trig identity:

$$\cos^2\theta = \frac{1}{2} + \frac{\cos(2\theta)}{2}.$$

$$= r^2 \int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{\cos(2\theta)}{2} \right) d\theta .$$

$$= r^2 \int_{-\pi/2}^{\pi/2} \frac{1}{2} d\theta + \frac{r^2}{2} \int_{-\pi/2}^{\pi/2} \cos(\underbrace{2\theta}_{\omega}) \, d\theta$$

$$= r^2 \frac{\theta}{2} \Big|_{-\pi/2}^{\pi/2} + \frac{r^2}{2} \frac{1}{2} \sin(2\theta) \Big|_{-\pi/2}^{\pi/2}.$$

$$= \frac{r^2}{2}\left(\frac{\pi}{2} - -\frac{\pi}{2} \right) + \frac{r^2}{4}\left(\sin(\pi) - \sin(-\pi) \right).$$

$$= \frac{r^2 \pi}{2} + \underbrace{\frac{r^2}{4}\left(0 - 0 . \right)}_{= 0} .$$

$$= \frac{\pi r^2}{2}$$

full

Area of Circle is: πr^2. //