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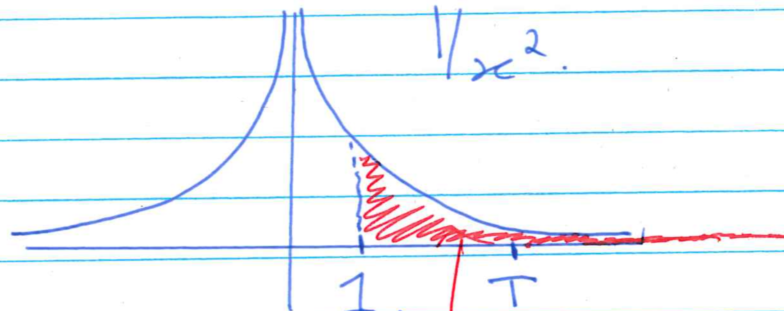
Nov. 30

- HW10 is Due
- HW11 (not for marks)
  - to be posted.

Exam: Dec. 14 12noon. in LSK.

Two more integrals:

1). $\int_1^{\infty} \frac{1}{x^2}\,dx$.



$= \lim_{T\to\infty} \int_1^T \frac{1}{x^2}\,dx$.

$= \lim_{T\to\infty} \int_1^T x^{-2}\,dx$

$= \lim_{T\to\infty} -x^{-1}\Big|_1^T = \lim_{T\to\infty} -\frac{1}{T} - -\frac{1}{1}$

$= 0 + 1 = 1$ //

2). $\int e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(x)$.

error function.

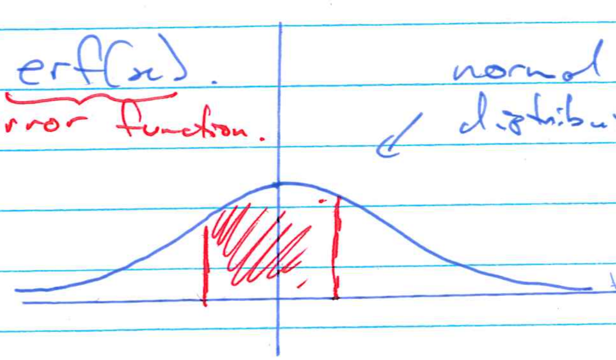
$u = -x^2$

$du = -2x\,dx$

$dx = \frac{-1}{2x}\,du$

$= -\frac{1}{2}\int e^u \frac{du}{x}$.

normal distribution.



Review:

Matt's ~~rules~~ guidlines for integrals.

Substitution:

- $\int \sin x \cos x \, dx$. (derivative of $\sin x$.)

Want: $u = \sin x$
$du = \cos x \, dx$.

- $\int (x^2+1)^2 \, x \, dx$ (kind of) derivative of x^2+1.

want: $u = x^2 + 1$.
$du = 2x \, dx$

Good choices for u include:

- Things insides things
    - $\int \sin(\underbrace{3x}_{u}) \, dx$.
- Things inside square roots
    - $\int x \sqrt{\underbrace{x^2+1}_{u}} \, dx$.
- Denominators:
    - $\int \frac{1}{\underbrace{4-3x}_{u}} \, dx$.

Integration by Parts:

$$\int u\,dv = uv - \int v\,du.$$

We differentiate u.
and integrate dv.

Choose u and v to make the integral easier.

Ex: $\int \underbrace{x}_{u} \underbrace{e^x\,dx}_{dv}$.

$u = x$
$du = dx$.
$dv = e^x dx$
$v = e^x$.

$= xe^x - \underbrace{\int e^x\,dx}_{\text{easier.}}$.

$= \underline{xe^x - e^x + C}$.

- e^x, $\sin x$, $\cos x$ – don't change much.
- x, x^2, x^3, gets easier with diff
- $\ln x$ gets easier with diff.

Ex: Sometimes these integrals are tricky:

$\int \underbrace{\sin x}_{u} \underbrace{\cos x\,dx}_{dv}$

$u = \sin x$ $\quad dv = \cos x\,dx$
$du = \cos x\,dx$ $\quad v = \sin x$.

$= \sin^2 x - \underbrace{\int \sin x \cos x\,dx}_{\text{move to left side}}$.

: $2\int \sin x \cos x\,dx = \dfrac{\sin^2 x}{2}$.

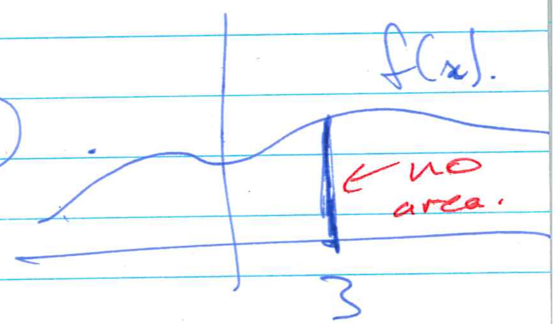
$$\int \sin x \cos x \, dx + \int \sin x \cos x \, dx = \sin^2 x$$

$$2 \int \sin x \cos x \, dx = \sin^2 x$$

$$\int \sin x \cos x \, dx = \frac{\sin^2 x}{2} + C.$$

These are indefinite.
Don't forget about definite:

- $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a).$
- $\displaystyle\int_3^3 f(x)\,dx = 0.$



- $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$
- $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx.$

$$-\sqrt{3} \int \cos x \, dx = -\sqrt{3} \sin x + C.$$

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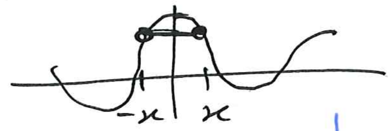
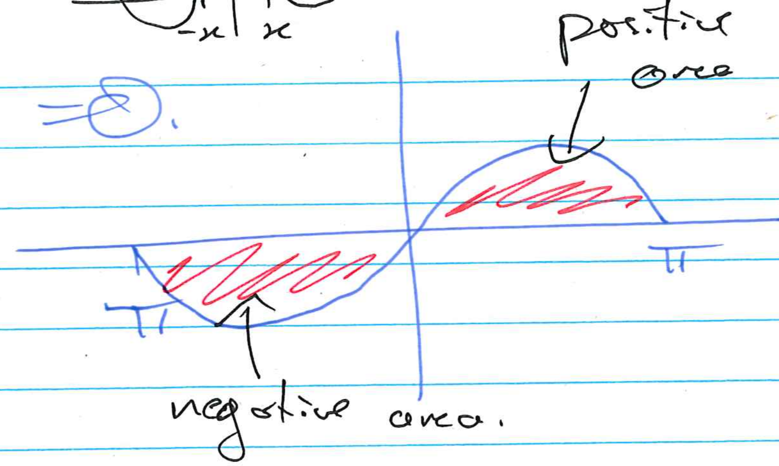


$$\int_{-\pi}^{\pi} \sin x \, dx = 0.$$

↑ odd.



$f(-x)$

$$\int_{-14}^{14} \underbrace{x^2 \cdot \cos x \cdot e^{x^2} \cdot x}_{\text{odd.}} \, dx = 0.$$

$$f(-x) = -f(x)$$

$$\int_{-1}^{1} x e^{x^2} \, dx$$

Sub. $\begin{cases} u = x^2. \\ du = 2x\,dx. \end{cases}$

$\frac{du}{2x} = dx.$

$f(x) = x e^{x^2}$ odd.

$f(-x) = -x e^{(-x)^2}$

$= -\left(x e^{x^2}\right)$

Wed:
- Integration word problems.
- Related Rates.
- Riemann Sums.