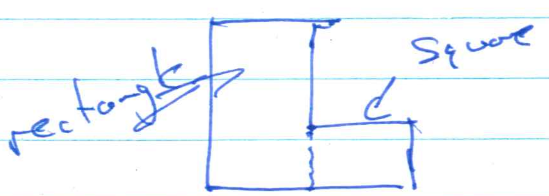
Nov. 4

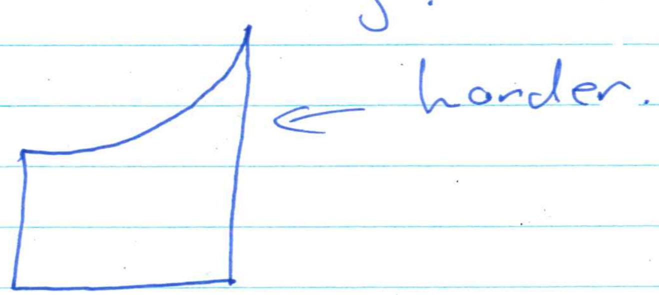
- Labs this week to take up Midterm.
- Midterm Solutions to be posted.
- No HW this week
- Quiz #4 Fri. Nov. 13
  - will post materials.

Integrals: (Part II).

Lets say we want to find the area of some shape:

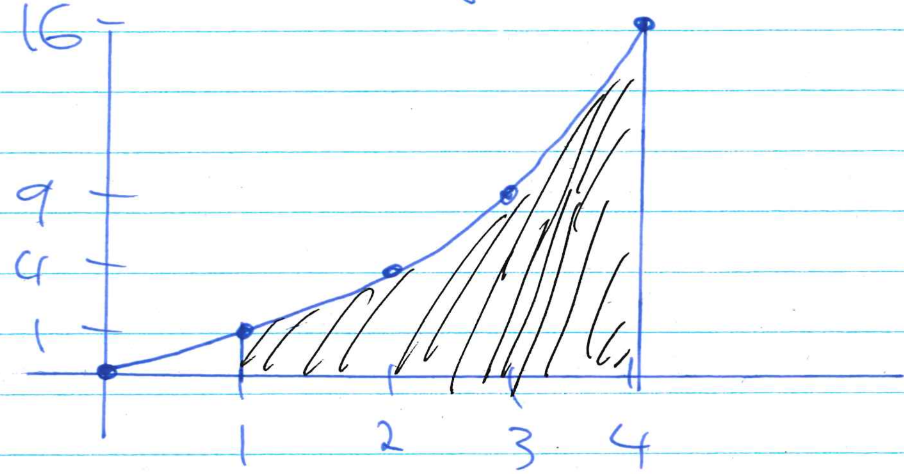
Some Shapes are easy.





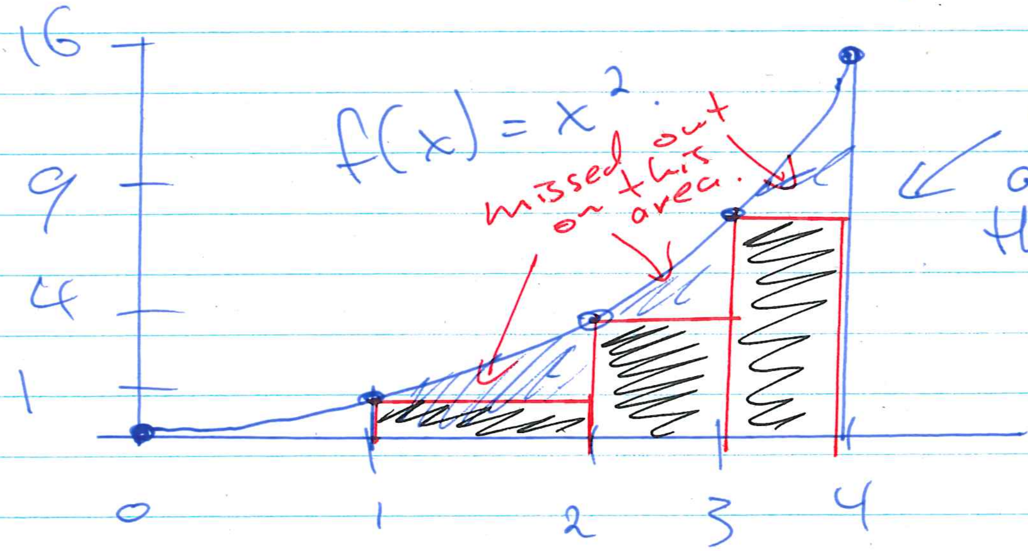
②

Let us try to get an idea of the area under the curve $y = x^2$ from $x=1$ to $x=4$,



Now computing this area is hard because of the curved line. If only we could introduce some rectangles.

Idea! ~~Introduce~~ Introduce some rectangles.



Can find area of these rectangles.

③

Area of rectangles.

$$1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1.$$

$$= 1 + 4 + 9 = 14.$$

$$\left(\sum_{\substack{i=1 \leftarrow \text{min} \\ \uparrow \text{counter}}}^{3 \leftarrow \text{max.}} i^2 \cdot 1 \quad \underline{\text{Sigma}} \ \underline{\text{notation.}} \right)$$

$$\left(\sum_{i=1}^{3} i^2 \cdot 1 = 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1 \right)$$

$$\left(\underline{\text{Ex!}} \ \sum_{i=1}^{7} i^2 \cdot 1 = 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1 + 4^2 \cdot 1 + 5^2 \cdot 1 + 6^2 \cdot 1 + 7^2 \cdot 1. \right)$$

$$\left(\sum_{i=2}^{4} i^2 \cdot 1 = 2^2 \cdot 1 + 3^2 \cdot 1 + 4^2 \cdot 1. \right)$$

<u>Clicker Q</u>: Is our estimate of 14 an

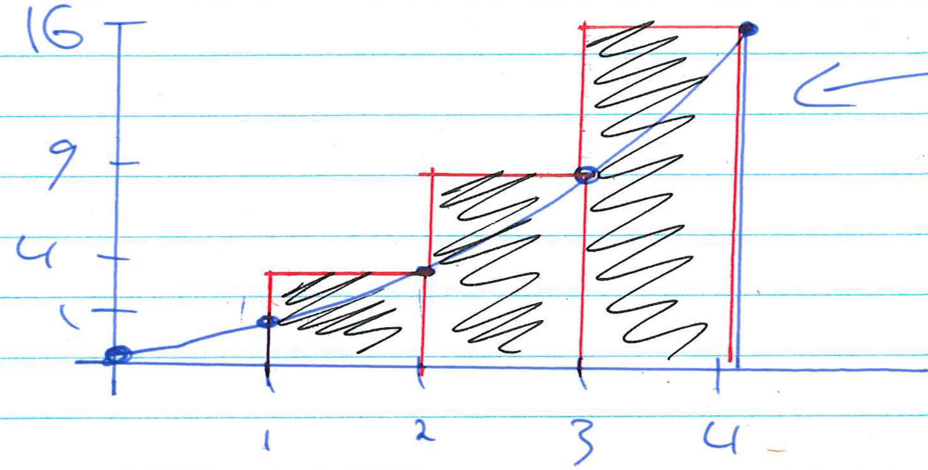
A) overestimate

→ B) underestimate

C) exactly right.

4

For an overestimate:



right endpoints

Area of these rectangles: overestimate

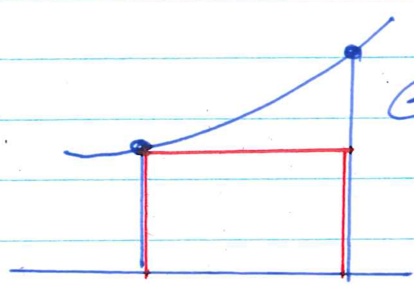
$$2^2 \cdot 1 + 3^2 \cdot 1 + 4^2 \cdot 1$$

$$= 4 + 9 + 16 = 29.$$

So the area we want is between 14 and 29.

pretty poor but it is something.

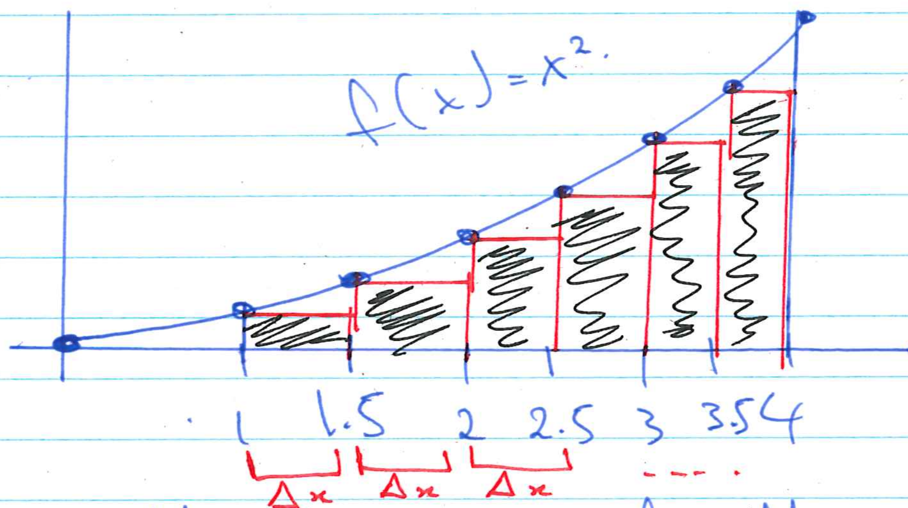
left endpoint.



(5)

To make this better we can use <u>more</u> rectangles:

Let's use left ~~t~~ endpoints.



$$\frac{4-1}{6}=\frac{1}{2} = \Delta x.$$

So the area of the rectangles is:

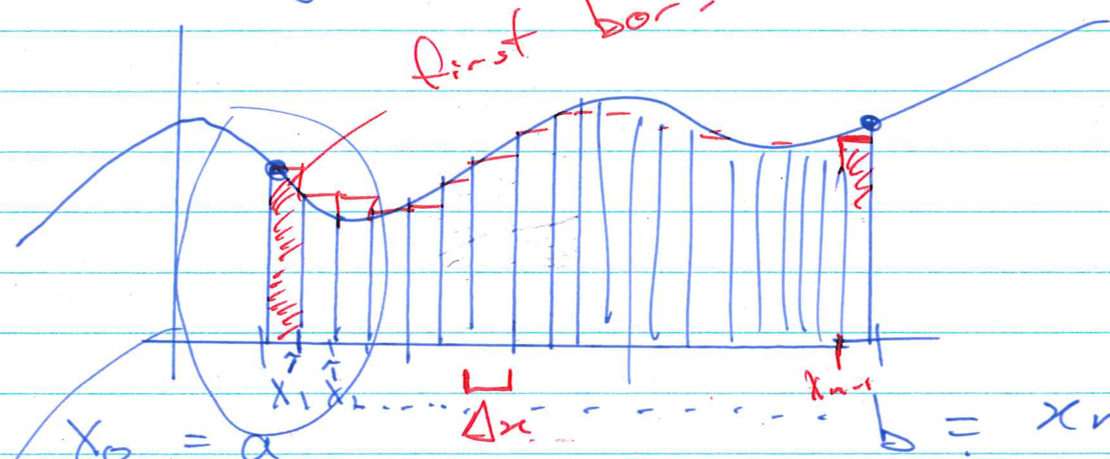
Calculator

$$1^2\cdot\left(\frac{1}{2}\right) + (1.5)^2\left(\frac{1}{2}\right) + 2^2\left(\frac{1}{2}\right) + (2.5)^2\left(\frac{1}{2}\right) + 3^2\left(\frac{1}{2}\right) + (3.5)^2\left(\frac{1}{2}\right),$$

$$= 17.375.$$

(2)

In general,



Cut the interval $[a,b]$ into n bars.

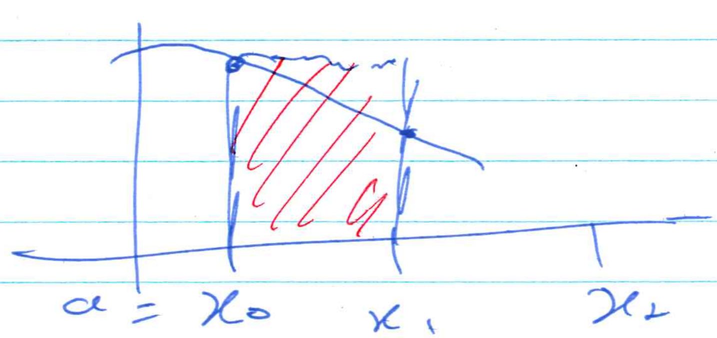
$$\Delta x = \frac{b-a}{n}$$

Area of all rectangles ~~this~~:
(left endpoint).

Clicker Q: Area of first bar

A) $1 \cdot \Delta x$ B) $1^2 \cdot \Delta x$

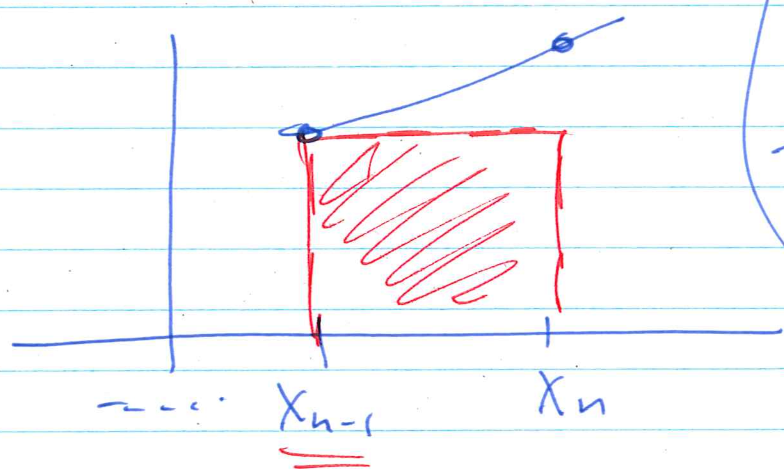
→ C) $f(x_0)\Delta x$ D) $f(x_1)\Delta x$.



All the bars: (left)

$$f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_{n-2})\Delta x + f(x_{n-1})\Delta x.$$

The last bar:

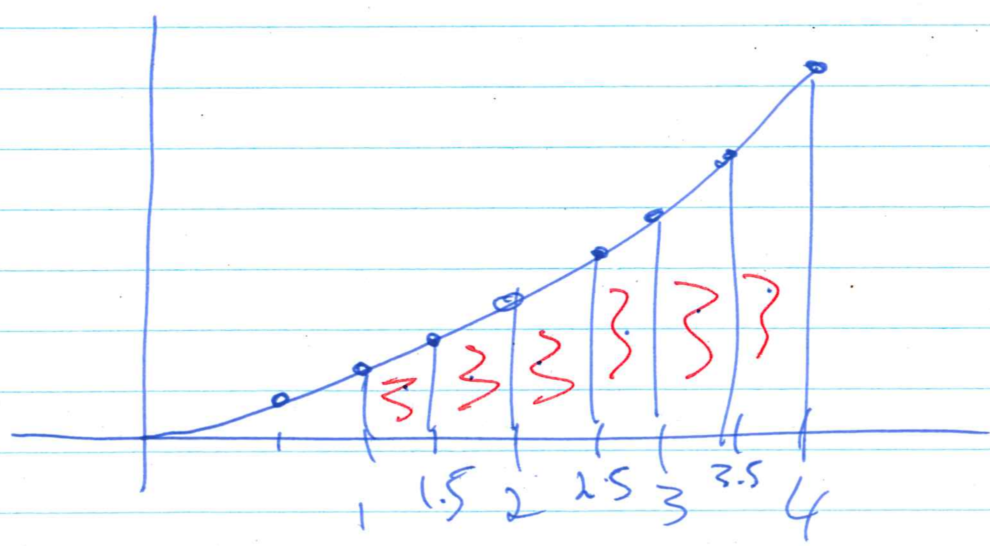


$\left(x_n - \Delta x \quad \cancel{x_n - \Delta x} \right)$

$n = 6 \quad x_n = x_6$

$n-1 = 5 \quad x_{n-1} = x_5.$

In the previous:



$\Delta x = 1/2$

$n = 6.$

$x_4 = 3$

$x_5 = 3.5$

$x_6 = 4.$

$x_0 = 1 \quad x_1 = 1.5 \quad x_2 = 2 \quad x_3 = 2.5.$

8

Right endpoints:

$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$
$+ \cdots + f(x_{n-1})\Delta x + f(x_n)\Delta x$

Using sigma notation.

- $\displaystyle\sum_{i=1}^{n} f(x_i)\,\Delta x.$ (right).

- $\displaystyle\sum_{i=0}^{n-1} f(x_i)\,\Delta x$ (left).

Idea will be to take $n \to \infty$.