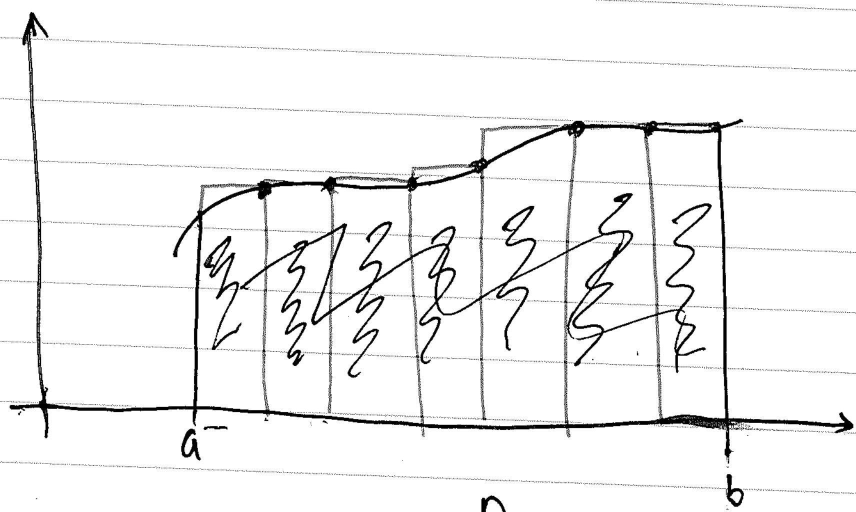
DEFINITE INTEGRAL

$f(x) \qquad a \le x \le b$



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x$$

$$\Delta x = \frac{(b-a)}{n}$$

x_i^* can be

- left end point
- right end point
- mid point

of the interval $[x_{i-1}, x_i]$.

x_{i-1}

Applications

~~Find~~ This ~~can~~ kind of limits are used in finding

- volume of a solid
- distance traveled by car
- length of curves

Definition: Definite integrals

$f(x)$, $[a, b]$,

$$\boxed{\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x}$$

Notations

$\int$ — integral sign

$f(x)$ — integrand

a and b are called the limits of integration

a - lower limit

b - upper limit

dx - indicates that the independent variable is x.

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x$$ - Riemann Sum

The process of calculating an integral is called integration

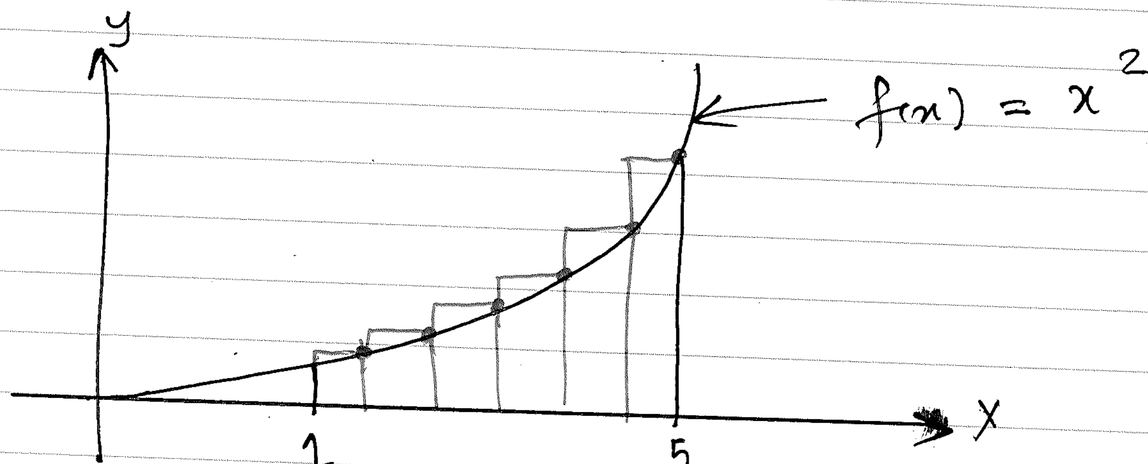
Example

Express
$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 \Delta x$$

as an integral over the interval $[1, 5]$

Solution

$f(x) = x^2$, $a = 1$, $b = 5$



$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 \Delta x = \int_1^5 x^2\, dx$$

Example

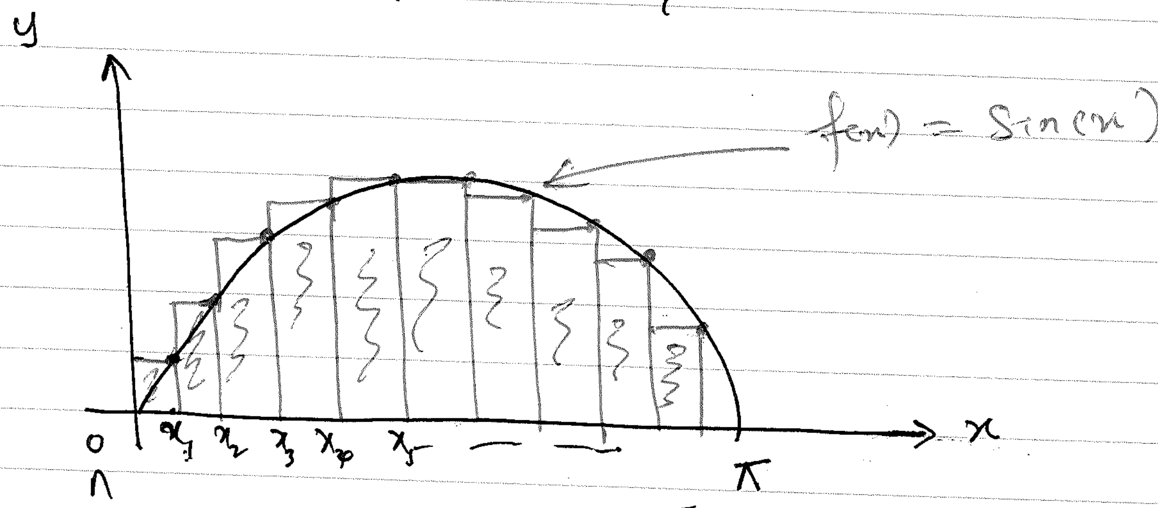
Express

$$\lim_{n \to \infty} \sum_{i=1}^{n} \sin(x_i)\,\Delta x$$

as an integral on the interval $[0, \pi]$

Solution

$f(x) = \sin(x)$, $a = 0$, $b = \pi$



$$\lim_{n \to \infty} \sum_{i=1}^{n} \sin(x_i)\,\Delta x = \int_0^{\pi} \sin(x)\,dx$$

Properties of sigma notation and integrals

(i) $$\boxed{\sum_{i=1}^{n} k = nk}$$

Check

$$\sum_{i=1}^{4} 5 = 5 + 5 + 5 + 5 = \underline{\underline{4\times 5}}$$

$$\boxed{\int_a^b k\,dx = \underline{\underline{k(b-a)}}}$$

$5\,(4-1)$

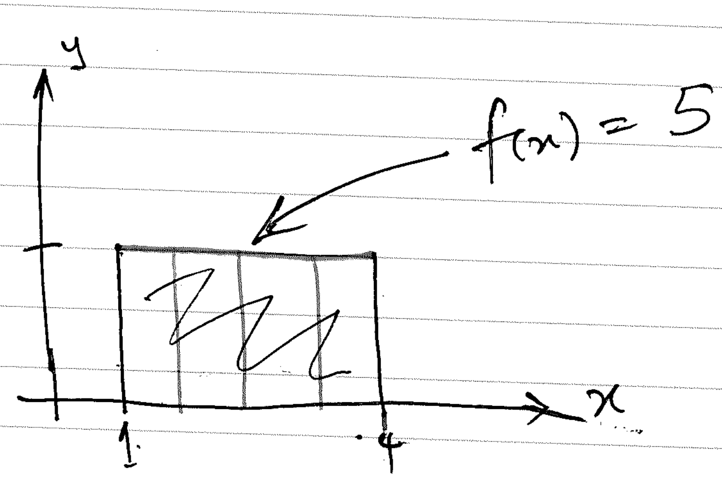
Example

$$\int_1^4 5\,dx$$

$f(x) = 5$

$a = 1$

$b = 4$



$$\text{Area} = L \times b = (4-1) \times 5 = 3\times 5 = \underline{\underline{15}}$$

$$\int_1^4 5\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} 5\,\Delta x \qquad a \times 1 = a$$

$$\Delta x = \frac{(b-a)}{n} = \frac{(4-1)}{n} = \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 5\left(\frac{3}{n}\right) = \cancel{5\left(\frac{3}{n}\right)}$$

$$= \lim_{n\to\infty} 5\left(\frac{3}{n}\right)\left(\underbrace{\sum_{i=1}^{n} 1}_{n}\right)$$

$$= \lim_{n\to\infty} 5\left(\frac{3}{\cancel{n}}\right)\cdot \cancel{n} = \lim_{n\to\infty} 5(3) = 5(3)$$

$$= \underline{\underline{5(4-1)}}$$

11

$$\boxed{\sum_{i=1}^{n} k x_i = k \sum_{i=1}^{n} x_i}$$

$$\sum_{i=1}^{n} 5\left(\frac{3}{n}\right) = 5\left(\frac{3}{n}\right)\sum_{i=1}^{n} 1$$

$$\boxed{\int_a^b k f(x)\, dx = k \int_a^b f(x)\, dx}$$

(iii) $$\boxed{\sum_{i=1}^{n}(x_i+y_i)=\sum_{i=1}^{n}x_i+\sum_{i=1}^{n}y_i}$$

$$\boxed{\int_a^b\big(f(x)+g(x)\big)\,dx=\int_a^b f(x)\,dx+\int_a^b g(x)\,dx}$$

$$\int_1^5(x^2+x^3)\,dx=\int_1^5 x^2\,dx+\int_1^5 x^3\,dx$$

(iv) $$\boxed{\sum_{i=1}^{n}(x_i - y_i) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i}$$

$$\int_a^b \left(f(x) - g(x)\right) dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

Example

$$\int_4^7 (\sin(x) + x^4)\,dx$$

$$= \int_4^7 \sin(x)\,dx + \int_4^7 x^4\,dx$$