Oct. 19.

- HW5 is due
- HW6 to be posted due Monday.
- Quiz #3 Friday
  - Week 5 and 6 material
    - tangent lines, limit definition, derivative rules.
- Midterm: Nov. 2
- Vote!

Last class product rule:

$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{d}{dx}\left(f(x)\right)g(x) + f(x)\frac{d}{dx}\left(g(x)\right)$$

$$= f'(x)g(x) + f(x)g'(x)$$

Similar to product rule is Quotient Rule:

Let's say we want the derivative of

$$h(x) = \frac{f(x)}{g(x)}.$$

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(For example: $h(x) = \dfrac{x}{x-1}$.)

Quotient Rule:

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

"low d-high minus high d-low, Square the bottom and away we go".

Ex: $h(x) = \dfrac{x}{x-1} = \dfrac{f(x)}{g(x)}$.

$f(x) = x$, $f'(x) = 1$
$g(x) = x-1$, $g'(x) = 1$.

$$h'(x) = \frac{1\cdot(x-1) - x\cdot 1}{(x-1)^2}.$$

$$= \frac{x-1-x}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2}.$$

as expected?

③

Example: $h(x) = \dfrac{\sin x}{\cos x} = \tan x$.

Find $h'(x)$.

$f(x) = \sin x$, $f'(x) = \cos x$

$g(x) = \cos x$, $g'(x) = -\sin x$.

$$h'(x) = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \quad (= \sec^2 x.)$$

So, $\dfrac{d}{dx}(\tan x) = \dfrac{1}{\cos^2 x}$.

(4)

You may need to combine ~~rule~~ rules.

$$h(x) = \frac{xe^x}{2x+1}$$ Find $h'(x)$.

Product rule within quotient rule.

$$h'(x) = \frac{(x\cdot e^x)'(2x+1) - xe^x(2x+1)'}{(2x+1)^2}$$

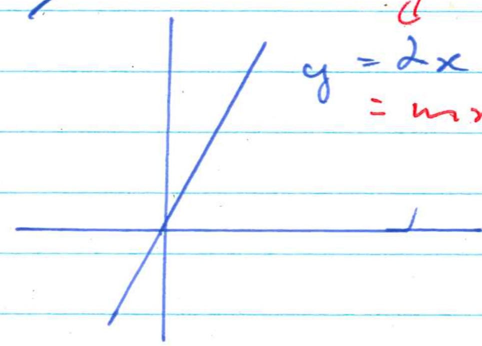
$$= \frac{(xe^x + e^x)(2x+1) - xe^x\cdot 2}{(2x+1)^2}$$

aside: $(2x^1)'$

slope 2.

$y = 2x$

$= mx + b$.



Simplification is not necessary.

$$= \frac{2x^2e^x + \cancel{2xe^x} + xe^x + e^x - \cancel{2xe^x}}{(2x+1)^2}$$

$$= \frac{2x^2e^x + xe^x + e^x}{(2x+1)^2}$$

There is one more differential rule we need. It happens to be the most useful/important.

Chain Rule (§ 2.4)

Suppose $h(x) = \sin(x^2)$

This is a composition of x^2 and $\sin x$.

Clicker Q: Can $h'(x) = \cos(x^2)$

A) Yes
B) No
C) I don't know

but at $x=0$ $\cos(0^2) = \cos(0) = 1$.

Could it be $\cos(2x)$?
but $\cos(2 \cdot 0) = 1 \neq 0$.

Turns out the derivative is

$$h'(x) = 2x\cos(x^2).$$

Chain Rule: Let $h(x) = f(g(x))$.

then $h'(x) = f'(g(x)) \cdot g'(x)$

or/ $y = f(u)$, $u = g(x)$

then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$.

"Take the derivative of the outside, leave the inside the same, multiply by the derivative of the inside."

So, in our example

$$h(x) = \sin(x^2) = f(g(x)).$$

outside

$\rightarrow f(x) = \sin x$, $g(x) = x^2$.

inside.

$f'(x) = \cos x$ $\quad g'(x) = 2x$.

$$f'(g(x)) = f'(x^2) = \cos(x^2)$$

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Now, $h'(x) = f'(g(x)) \cdot g'(x)$

$= \cos(x^2) \cdot 2x$

$= 2x\cos(x^2)$.

Clicker Q: What is $\frac{d}{dx}(e^{-x})$.

26% A) e^{-x} → 43% B) $-e^{-x}$

2% C) e^{x} 26% D) $-e^{x}$.

$h(x) = f(g(x))$.

$f(x) = e^{x}$ $g(x) = -x$.

$f'(x) = e^{x}$

$f(g(x)) = f(-x) = e^{-x}$.

$f(2) = e^{2}$, $f(3) = e^{3}$.

$f(x+h) = e^{x+h}$, $f(-x) = e^{-x}$.

$\frac{d}{dx}(e^{-x}) = f'(g(x)) \cdot g'(x)$

$= e^{-x} \cdot (-1)$

$= -e^{-x}$.