① Oct. 2.

- HW3 Due Monday (MLC LSK ~~301~~ 300)
- Quiz # 2 Fri. Oct. 9
  - Week 4 Notes
  - Week 4 practice problems

  } limits.

- HW 2 returned (see solutions).

$$\sin\left(\frac{1}{x}\right) = 0 \overset{\text{steps?}}{\Longrightarrow} x = \frac{1}{n\pi}.$$

(see solutions)

Warm up: $\displaystyle \lim_{x\to -5} \frac{\frac{1}{x} + \frac{1}{5}}{5 + x}$.

try subsitution: "$\frac{0}{0}$".

Sub fails: Algebra → Cancel → Sub.

$$= \lim_{x\to -5} \frac{\frac{5+x}{5x}}{\frac{5+x}{1}} = \lim_{x\to -5} \frac{\cancel{(5+x)}}{5x} \cdot \frac{1}{\cancel{(5+x)}}$$

$$= \lim_{x\to -5} \frac{1}{5x}$$

$$= \frac{1}{5(-5)} = \frac{-1}{25}.$$

②

There are still a few types of limits we have yet to talk about.

Let us investigate the limit

$$\lim_{x \to 0^+} \frac{1}{x}.$$

If x is a small positive number then $1/x$ is a big positive number.

We write $\lim\limits_{x \to 0^+} \frac{1}{x} = \infty$.

y-value.

to mean: "$\frac{1}{x}$ gets big as x approaches zero from above/right".

Similarly

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

means: $\frac{1}{x}$ gets really big (but negative) as x approaches 0 from below/left.

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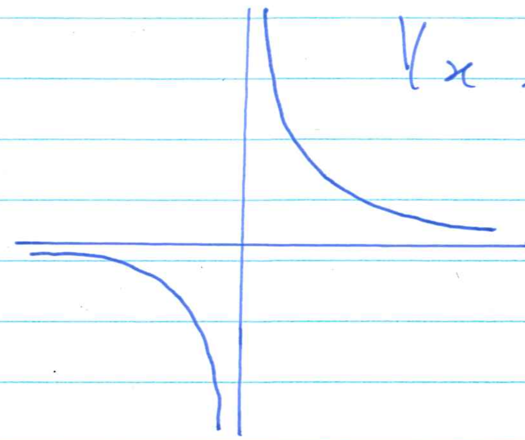
Note: $\pm\infty$ are not numbers.

Clicker Q: $\lim_{x \to 0} \frac{1}{x} = ?$

A) ∞
B) $-\infty$
C) $\pm\infty$
→ D) DNE.



(aside: $\lim_{x \to 0^+} \frac{1}{x} = \infty$

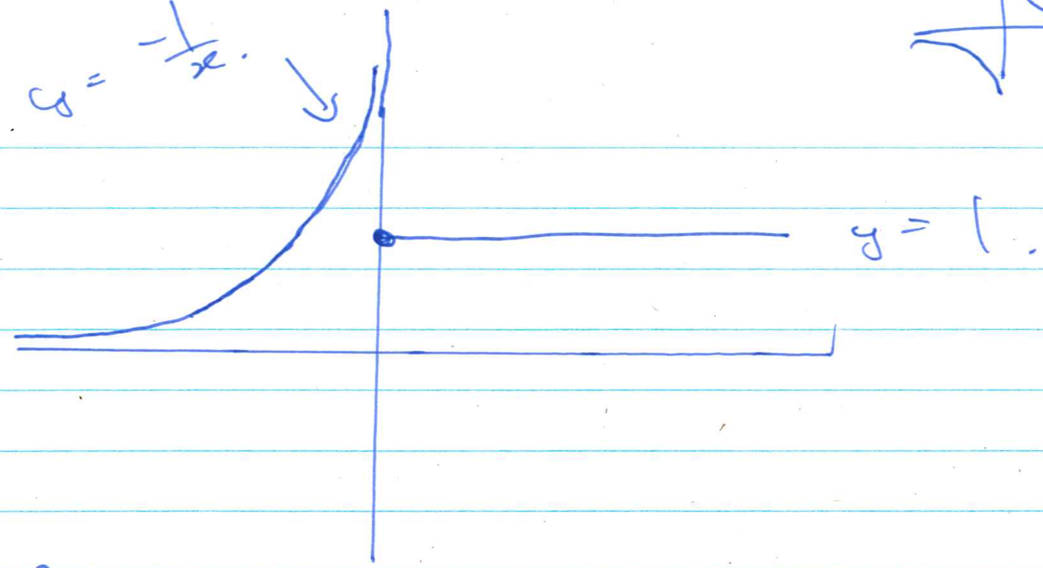
$\lim_{x \to 0^-} \frac{1}{x} = -\infty.$ } technically neither of these limits exist)

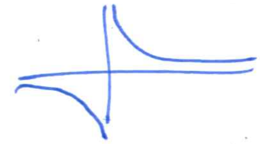
Clicker Q: Does this function have a vertical asymptote?

$$f(x) = \begin{cases} -1/x, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

A) Yes 38%
B) No . 62%

(4)





- $f(0) = 1$
- range f is all $\mathbb{R}$.

(A) Yes, the reason is that

- $\lim_{x \to 0^-} f(x) = \infty$.

$\Downarrow$

vertical asymptote.

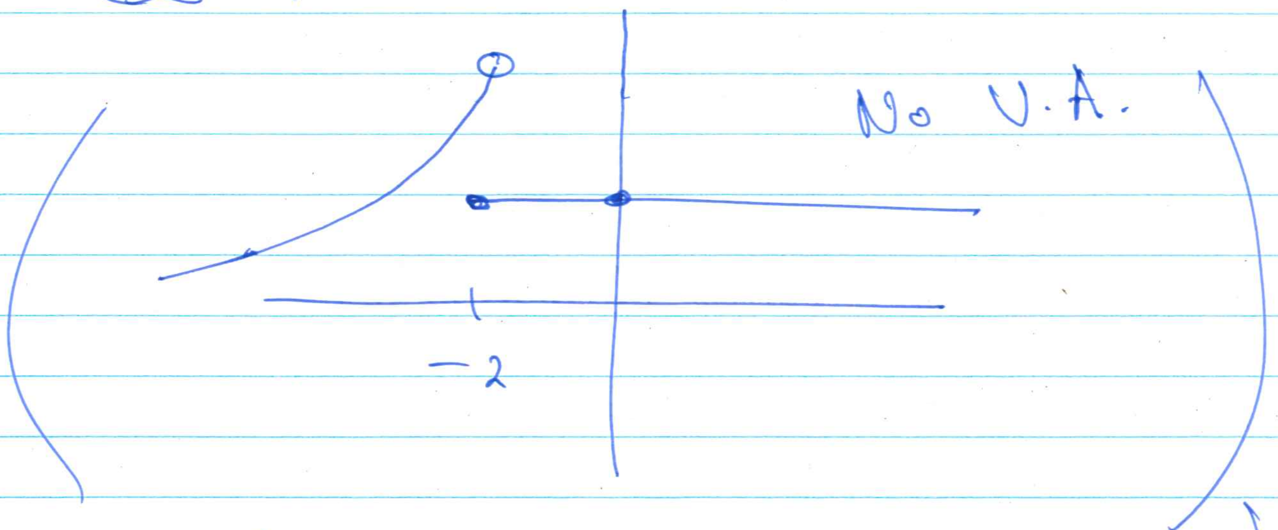
We say a function has a vertical asymptote at $x = a$ if

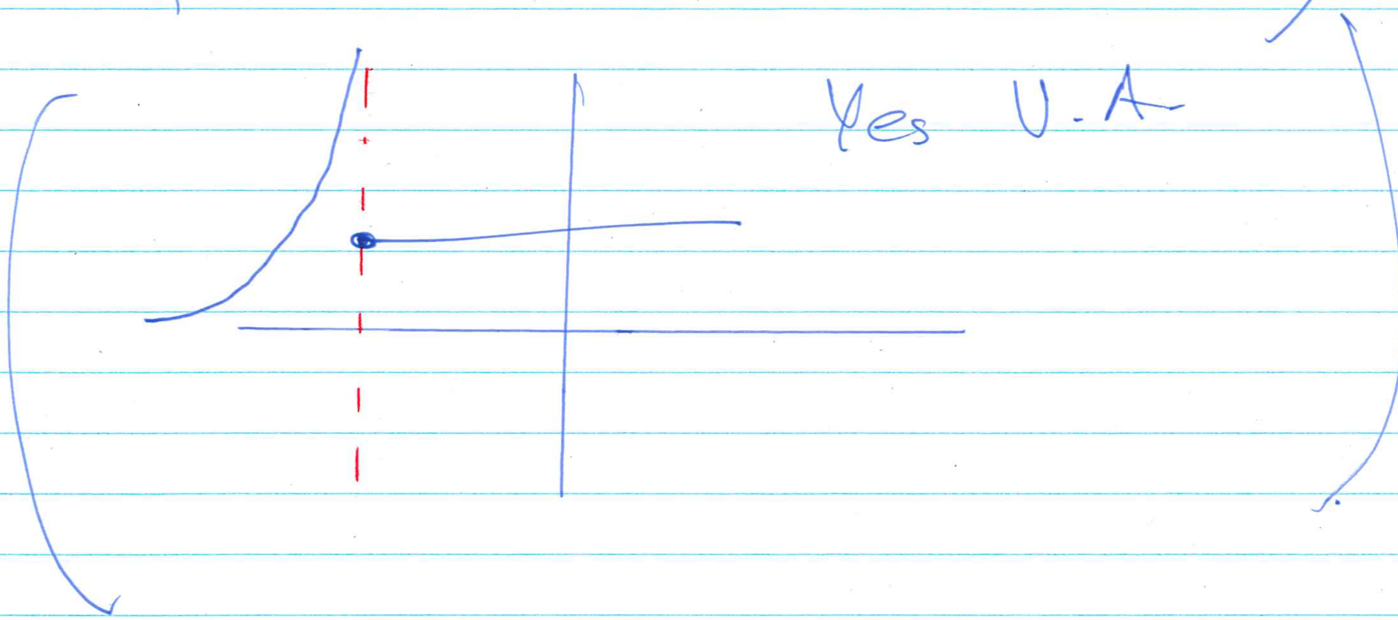
one of (or both)

- $\lim_{x \to a^-} f(x) = +\infty$ or $-\infty$
- $\lim_{x \to a^+} f(x) = +\infty$ or $-\infty$.

(5)

That is if either of the one sided limits is $+\infty$ or $-\infty$.





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Example: Find all vertical asymptotes of

$$f(x) = \frac{x+1}{x^2 - x - 2}$$

$$= \frac{x+1}{(x+1)(x-2)}.$$

Candidates for vertical asymptotes.

$x = -1$, $x = 2$.

Let's take the one sided limits at these points

First $x = -1$.

- $\lim_{x \to -1^-} \frac{(\cancel{x+1})}{(\cancel{x+1})(x-2)} = \lim_{x \to -1^-} \frac{1}{x-2} = -1/3.$
- Similarly, $\lim_{x \to -1^+} \frac{(\cancel{x+1})}{(\cancel{x+1})(x-2)} = -1/3.$

not $+\infty$ or $-\infty$.

$\Rightarrow$ no vertical asymptote at $x = -1$.

Now $x = 2$:

- $\displaystyle \lim_{x \to 2^-} \frac{(\cancel{x+1})}{(\cancel{x+1})(x-2)} = \lim_{x \to 2^-} \frac{1}{x-2}$.

$\left(\frac{+}{-}\right) \qquad = -\infty$.

- $\displaystyle \lim_{x \to 2^+} \frac{(x+1)}{(x+1)(x-2)} = \lim_{x \to 2^+} \frac{1}{x-2} \quad \left(\frac{+}{+}\right)$

$= +\infty$.

$\Rightarrow$ we have a V.A at $x = 2$.

By the way,