Oct-21

- HW4 Returned
- HW5 Solutions Posted
- HW6 Due Monday
- Quiz #3 Friday (up to & Product)

Chain Rule: (§2.4)

Example: $h(x) = \sqrt{x^2+1}$.
Find $h'(x)$.

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)) \cdot g'(x).$$

$$\sqrt{x^2+1} = f(g(x)).$$

$$f(x) = \sqrt{x}$$
$$g(x) = x^2 + 1.$$

$$f(x) = x^{1/2} \qquad f'(x) = \frac{1}{2} x^{-1/2}$$

$$g'(x) = 2x$$

$$h'(x) = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+1}}.$$

aside: $f(x) = \sqrt{x}$, $g(x) = x^2+1$.

$f(g(x)) = f(x^2+1)$

$= \sqrt{x^2+1}$.

Example:

$$\frac{d}{dx}\left[(2x^3+4)^7\right].$$

$f(x) = x^7$, $f'(x) = 7x^6$

$g(x) = 2x^3+4$, $g'(x) = 6x^2$.

$= f'(g(x)) \cdot g'(x)$

$= 7(2x^3+4)^6 \cdot 6x^2$.

$= 7 \cdot 6 (2x^3+4)^6 x^2$.

↑ Now let's take the derivative of this.

This is called the second derivative.

$h''(x) = \left(7 \cdot 6 (2x^3+4)^6 \cdot x^2\right)'$.

(3)

product rule

$$h''(x) = \left(42(2x^3+4)^6\right)' x^2 + 42(2x^3+4)^6 (x^2)'$$

chain Rule.

$$= 6 \cdot 42(2x^3+4)^5 \cdot 6x^2 \cdot x^2 + 42(2x^3+4)^6 \cdot 2x$$

$\left(42(\underbrace{2x^3+4}_{g(x)})^6\right)$ $\quad f(x) = 42x^6$, $\quad g(x) = 2x^3+4$.

$f'(x) = 6 \cdot 42 x^5$

$g'(x) = 6x^2$.

$f'(g(x)) \cdot g'(x) = 6 \cdot 42(2x^3+4)^5 \cdot 6x^2$.

$h'(x) = F(x) G(x)$

$F(x) = 42(2x^3+4)^6$

$G(x) = x^2$

$h''(x) = F'(x) G(x) + F(x) G'(x)$

Example: 1) $\sin(e^x)$

2) e^{2x}

3) e^{-x}.

Find the derivatives.

1) $\frac{d}{dx}\left(\sin(e^x)\right)$.

$= \cos(e^x) \cdot e^x$.

2) $e^{2x} = f(g(x))$.

$f(x) = e^x$, $f'(x) = e^x$

$g(x) = 2x$, $g'(x) = 2$.

$(e^{2x})' = e^{2x} \cdot 2$

$= 2e^{2x}$.

(5)

$$\left(e^{-x}\right)'$$

$f(x) = e^x$

$g(x) = -x$.

$f'(x) = e^x$ $\quad$ $g'(x) = -1$.

$$\left(e^{-x}\right)' = e^{-x} \cdot (-1)$$

$$= -e^{-x}.$$

Derivative of $\ln x$.

Notice $e^{\ln x} = x$.

Take the derivative of both sides:

$$\frac{d}{dx}\left(e^{\ln x}\right) = \frac{d}{dx}(x)$$

$$e^{\ln x}\frac{d}{dx}(\ln x) = 1$$

$$\frac{d}{dx}(\ln x) = \frac{1}{e^{\ln x}}.$$

$$(\ln x)' = \frac{1}{x}.$$

(6)

So $(\ln x)' = \frac{1}{x}$.

Ex: $\frac{d}{dx}\left[\ln(2x-1)\right]$

$f(x) = \ln x \qquad f'(x) = 1/x$

$g(x) = 2x-1 \qquad g'(x) = 2$.

$= f'(g(x)) \cdot g'(x)$

$= \frac{1}{2x-1} \cdot 2 = \frac{2}{2x-1}$.

Example: $\left[\ln\left(\sqrt{x-2}\right)\right]'$

$= \left[\ln\left((x-2)^{1/2}\right)\right]'$

$= \left[\frac{1}{2}\ln(x-2)\right]'$

$= \frac{1}{2}\left(\ln(x-2)\right)'$

$= \frac{1}{2}\cdot\frac{1}{x-2}\cdot 1 = \frac{1}{2(x-2)}$.