① Oct. 26.

- HW 6 Due Today.
- Labs for review this week
- Wed/Fri classes review.

Office Hours This Week:

Wed: 11-1pm MATX 1102
Ian → 1-2pm MATX 1118

Thurs: 3:30-5pm MATX 1102.

Fri: 1-3pm LSK 300C
Ian → 3-4pm LSK 300C.

MATX - Math Annex.

- No HW this week

Topics:
- Functions (trig. exp. log.)
- Limits
- Asymptotes
- Tangent lines / Limit Def
- Derivative Rules
  - power, product, quotient, chain
  - ~~ex~~ e^x, $\sin x/\cos x$, $\ln x$.

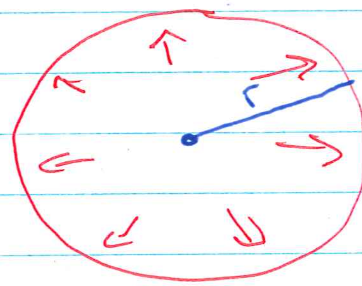
2

Where to find Questions?

- Examples done in class.
- HW problems.
- Quiz problems.
- Lab problems.
- Practice problems / review material.
- Check learning goals.

Related Rates (§ 2.6)

Example: The radius of a circular forest fire is increasing at a constant rate of 5 km/h. How fast is the area consumed by the fire increasing when the radius is 10 km.



A - Area
r - radius.
t - time.

What is the area of a circle?

$$A = \pi r^2 .$$

rate of change of radius.

Given: $\frac{dr}{dt} = 5 km/h.$

Required: $\frac{dA}{dt}$?

4

Try: $\frac{dA}{dr} = 2\pi r$.

Chain Rule: take $\frac{d}{dt}$ of both sides

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2).$$

$$\frac{dA}{dt} = \pi \frac{d}{dt}(r^2).$$

Clicker Q: What is $\frac{d}{dt}(r^2)$?

55% A) $2r$ ⟹ C) $2r\frac{dr}{dt}$

B) $2t$ D) No idea.

aside: $\frac{d}{dr}(r^2) = 2r$

$$\frac{d}{dt}\left((t^2+1)^2\right)$$

$$= 2(t^2+1)\cdot 2t.$$

⑤

$$\frac{d}{dt}\left[(t^2+1)^2\right].$$

inside : $r = t^2+1$.

$$\frac{d}{dt}(r)^2.$$

Chain!

$f(x) = (x^2+1)^2$

$u = x^2+1$.

$f(u) = u^2$.

derivative of f where u is variable

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}.$$

derivative of u where x is the variable.

$$\frac{df}{du} = 2u \qquad \frac{du}{dx} = 2x.$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = 2u \cdot 2x$$

$$= 2(x^2+1) \cdot 2x.$$

6

Chain Rule

$$\left((x^2+1)^2 \right)$$

$f(x) = x^2$ $\quad$ $f'(x) = 2x$

$g(x) = x^2+1$ $\quad$ $g'(x) = 2x$.

$$f'(g(x)) \cdot g'(x).$$

$$2(x^2+1) \cdot 2x.$$

So $\dfrac{d}{dt}(r^2)$

Chain Rule: $\dfrac{d}{dt}(r^2) = \dfrac{d(r^2)}{dr} \cdot \dfrac{dr}{dt}$.

$$= 2r \cdot \frac{dr}{dt}.$$

Back to the question:

$$A = \pi r^2.$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}.$$

using chain Rule.

$$\frac{dr}{dt} = 5\text{ km/h}.$$

Want $\frac{dA}{dt}$ when $r = 10\text{km}$

Just substitute and solve.

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}.$$

$$= \pi 2 \cdot (10\text{km})(5\text{km/h})$$

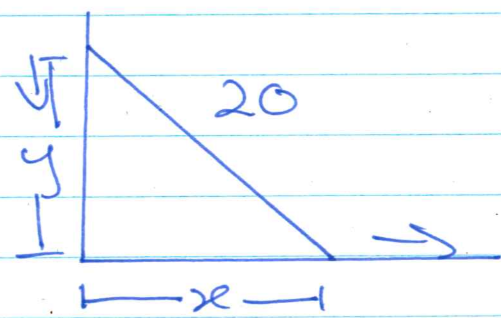
$$= 100\pi \text{ km}^2/\text{h}.$$

Example: A fallen tree (20m high) is resting against another tree.

The base slides at a rate of 3m/s.

How fast is the square decending when the base of the fallen tree is 12m from the base of the other?

1. Picture



2. Given: $\frac{dx}{dt} = 3\ m/s.$

Want: $\frac{dy}{dt}$? when $x = 12$.

3. Equation: $x^2 + y^2 = 20^2$.

4. Chain Rule:

$$\frac{d}{dt}\left(x^2 + y^2\right) = \frac{d}{dt}\left(20^2\right)$$

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0.$$

chain Rule

5. Solve / Substitute.

$$\cancel{2} y \frac{dy}{dt} = - \cancel{2} x \frac{dx}{dt}$$

$$\frac{dy}{dt} = - \frac{x}{y} \frac{dx}{dt}.$$

$$\frac{dy}{dt} = - \frac{(12)}{y} \cdot 3.$$

can find using.

$$x^2 + y^2 = 20^2.$$

$$12^2 + y^2 = 20^2.$$

$$\vdots$$

$$y = 16.$$

$$\frac{dy}{dt} = \frac{-12}{16} \cdot 3$$

$$= -\frac{9}{4} \text{ m/s}.$$