1 Oct. 30

- HW6 returned
  - no name?
- Midterm: Monday (10am).

Review:

What does it mean if a function has a horizontal asymptote $y = L$.

$\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

To find H.A.?

Consider $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$?

Example: $f(x) = \dfrac{x+1}{x^2-3}$.

$$\lim_{x \to \infty} \frac{x+1}{x^2-3} = \lim_{x \to \infty} \frac{x/x^2 + 1/x^2}{x^2/x^2 - 3/x^2}.$$

$$= \lim_{x \to \infty} \frac{1/x + 1/x^2}{1 - 3/x^2}.$$

$$= \frac{0+0}{1-0} = 0.$$

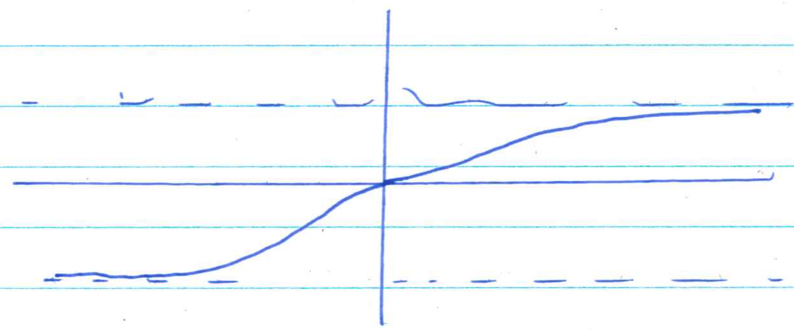
$\Rightarrow$ H.A. $y = 0$.

②

Similarly $\displaystyle\lim_{x\to-\infty} \frac{\overset{\to 0}{1/x} + \overset{\to 0}{1/x^2}}{1 - \underset{\to 0}{3/x^2}}$

$$= \frac{0+0}{1-0} = 0.$$

$\Rightarrow \quad y = 0.$



two H.asy.

Find the derivative of

$$f(x) = \underbrace{\ln}_{\text{outside.}} \big(\underbrace{\sqrt{2x+3}}_{\text{inside.}}\big).$$

$$f'(x) = \frac{1}{\sqrt{2x+3}} \left(\sqrt{2x+3}\right)'.$$

$$= \frac{1}{\sqrt{2x+3}} \frac{1}{2}(2x+3)^{-1/2} (2x+3)'$$

$$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{2} (2x+3)^{-1/2} \cdot 2.$$

③

$$= \frac{1}{\sqrt{2x+3}} \cdot \frac{1}{\sqrt{2x+3}}.$$

$$= \frac{1}{2x+3}.$$

Use log rules:

$$f(x) = \ln\left(\sqrt{2x+3}\right)$$

$$= \ln\left((2x+3)^{1/2}\right).$$

$$= \frac{1}{2}\ln(2x+3).$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{2x+3} \cdot (2x+3)'$$

$$= \frac{1}{\cancel{2}} \frac{1}{2x+3} \cdot \cancel{2}$$

$$= \frac{1}{2x+3}.$$

Ex:

Find the equation of the tangent line to

$$f(x) = \ln\left(\sqrt{2x+3}\right)$$

at $x = 0$.

§ Find slope - Find derivative.

$$f'(x) = \frac{1}{2x+3}.$$

So the slope at $x = 0$ is:

slope at $x=0$.

$$f'(0) = \frac{1}{0+3} = 1/3.$$

$$m = 1/3.$$

point slope.

$$y - y_1 = m(x - x_1)$$

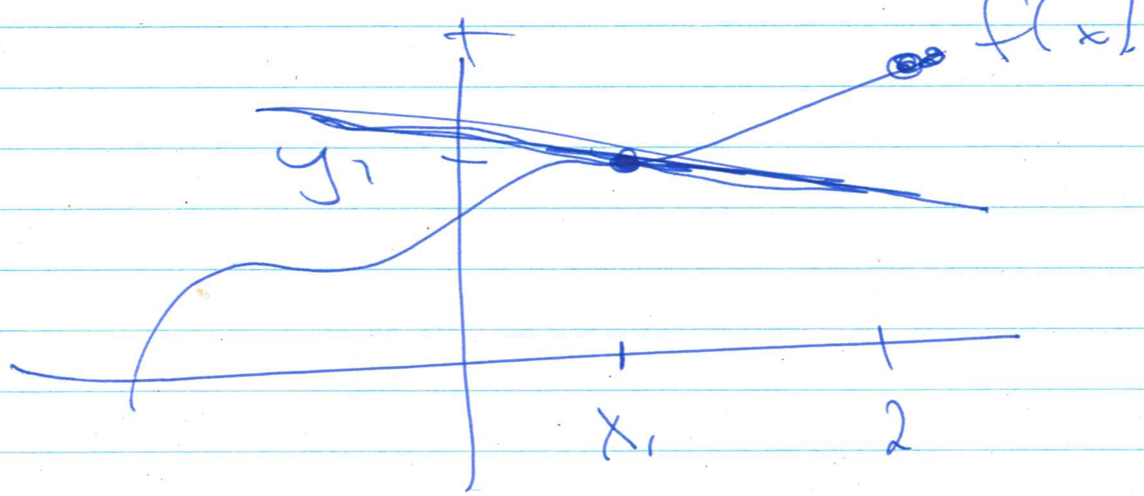
$$y - y_1 = 1/3(x - x_1).$$

$$\begin{cases} x_1 = 0. \\ y_1 = f(x_1 = 0). \end{cases}$$

(5)

$$f(0) = \ln\sqrt{3} = \underset{\overset{\|}{y_1}}{\frac{1}{2}\ln 3}.$$

$$y - \frac{1}{2}\ln 3 = \frac{1}{3}(x - 0).$$



⑥

9. Find all $x \in [0, \pi)$ such that

$$g(x) = \cos(\underbrace{2x}_{\text{inside.}}) - 2x.$$

has horizontal tangent lines.

Want to find x where slope is zero.

$$g'(x) = -\sin(2x) \cdot 2 - 2.$$

Want $g'(x) = 0$.

So we solve.

$$-2\sin(2x) - 2 = 0.$$

$$-2\sin(2x) = 2.$$

$$\sin(2x) = -1.$$

$$\sin(u) = -1.$$

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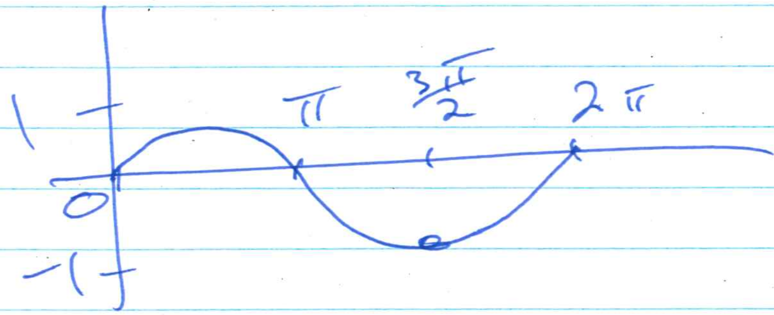


$$\sin(u) = -1$$

$$u = 3\pi/2$$

$$2x = u = \frac{3\pi}{2}$$

$$2x = \frac{3\pi}{2}$$

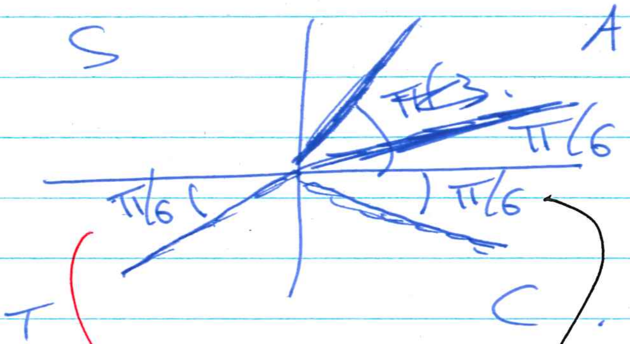
$$x = \frac{3\pi}{4} \in [0, \pi)$$

1. b) $f(x) = (-2x^2 + 3x) \cdot (x+1)^{-1}$

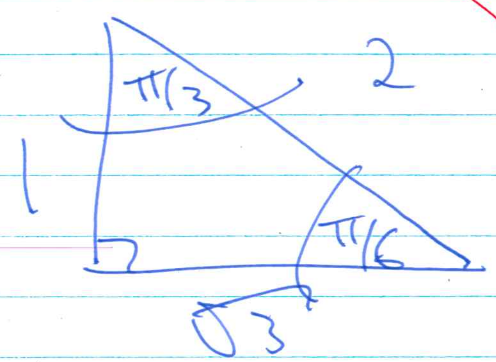
$$f'(x) = (-4x+3) \cdot (x+1)^{-1} + (-2x^2+3x)(-1)(x+1)^{-2} \cdot 1$$

$$= \frac{-4x+3}{x+1} - \frac{-2x^2+3x}{(x+1)^2}$$

$$\left(\sin(u) = -1/2 \right)$$



$$\sin(\pi/6) = \frac{1}{2}$$



SOH.

$$\pi + \pi/6 = 7\pi/6$$

$$2\pi - \pi/6 = 11\pi/6$$

8a).

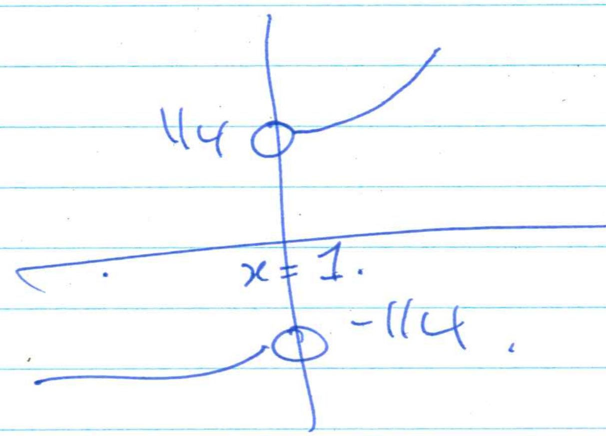
$$\lim_{x \to 1} \frac{|x-1|}{x^2+2x-3}.$$

$$|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1. \end{cases}$$

Find:

- $$\lim_{x \to 1^+} \frac{|x-1|}{x^2+2x-3}$$

- $$\lim_{x \to 1^-} \frac{|x-1|}{x^2+2x-3}$$



$$\lim_{x \to 1^+} \frac{(x-1)}{x^2+2x-3}.$$

$$= \lim_{x \to 1^+} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \lim_{x \to 1^+} \frac{1}{x+3}$$

$$= \frac{1}{4}.$$

- $$\lim_{x \to 1^-} \frac{-\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \frac{-1}{4}.$$

$$\Longrightarrow \text{D.N.E.}$$

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Limit Def:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$