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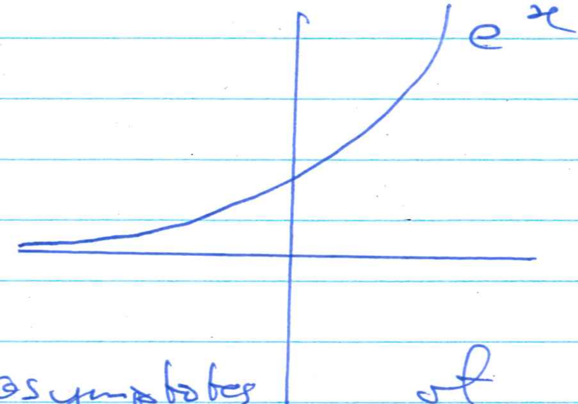
- HW 3 Solutions posted
- HW4 Due Wed.
- Quiz # 2 Friday.

$$\lim_{x \to -\infty} e^x = 0$$



Find the horizontal asymptotes of

$$g(x) = \frac{-2}{e^x + 3}.$$

- $\lim_{x \to \infty} \frac{-2}{e^x + 3} = 0 \Rightarrow$ H.A. at $y = 0$.
- $\lim_{x \to -\infty} \frac{-2}{e^x + 3} = \frac{-2}{0 + 3} = -2/3.$

$\Rightarrow$ H.A. at $y = -2/3$.

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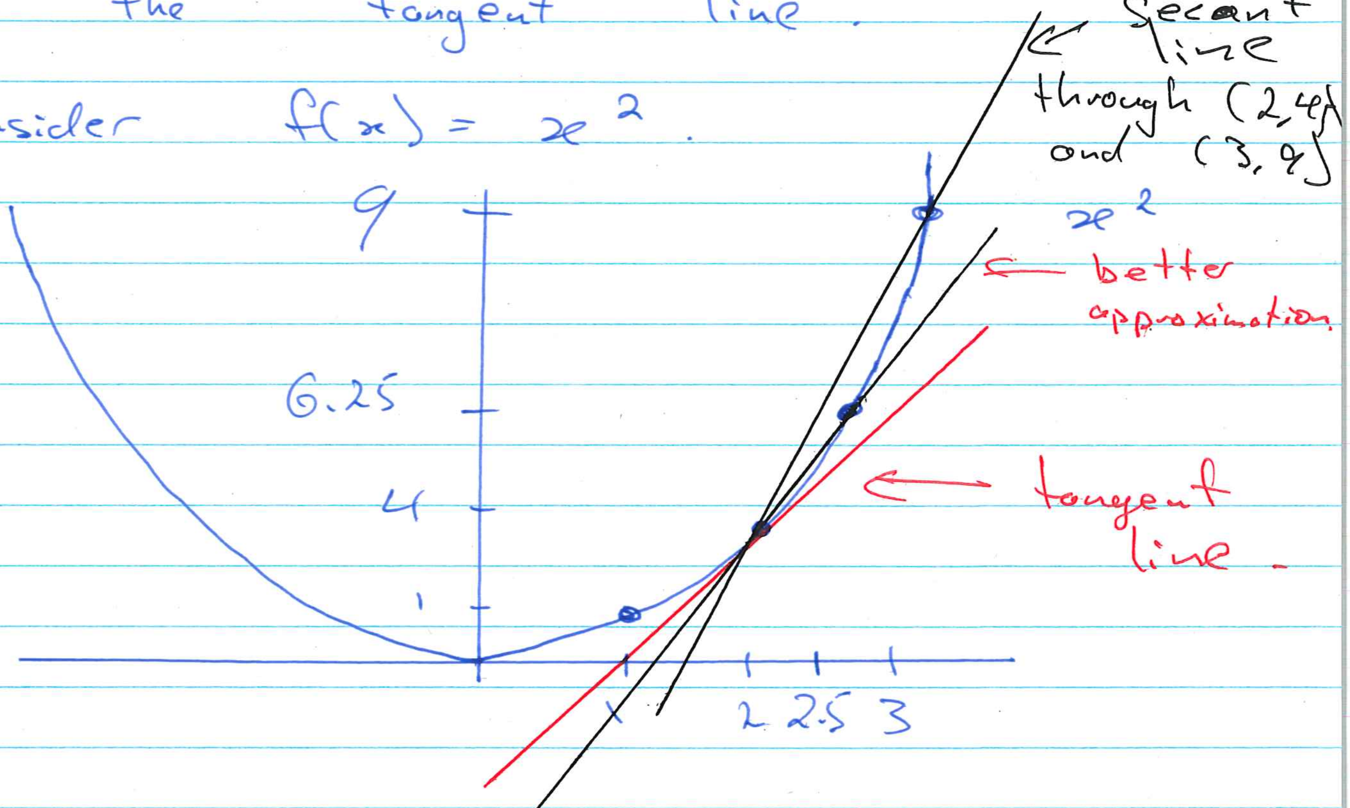
§ 2.0 and § 2.1

Definition of the Derivative.

We want to find the slope of the tangent line.

Consider $f(x) = x^2$.



Finding the slope of the tangent line is hard since we only have one point.

Easier is finding the slope of the secant line.

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We have two points.

$$m_{sec} = \frac{9-4}{3-2} = \frac{5}{1} = 5.$$

Crude approximation of the $m_{tangent}$.

A better approximation would be to use the secant line passing through $(2,4)$ and $(2.5, 6.25)$.

$$m_{sec} = \frac{6.25-4}{2.5-2} = \frac{2.25}{0.5} = 4.5.$$

better.

Even better is ~~(2.4)~~ $(2.1, 4.41)$.

$$m_{sec} = \frac{4.41-4}{2.1-2} = \frac{0.41}{0.1} = 4.1$$

even better.

To find the tangent line exactly we take the limit.

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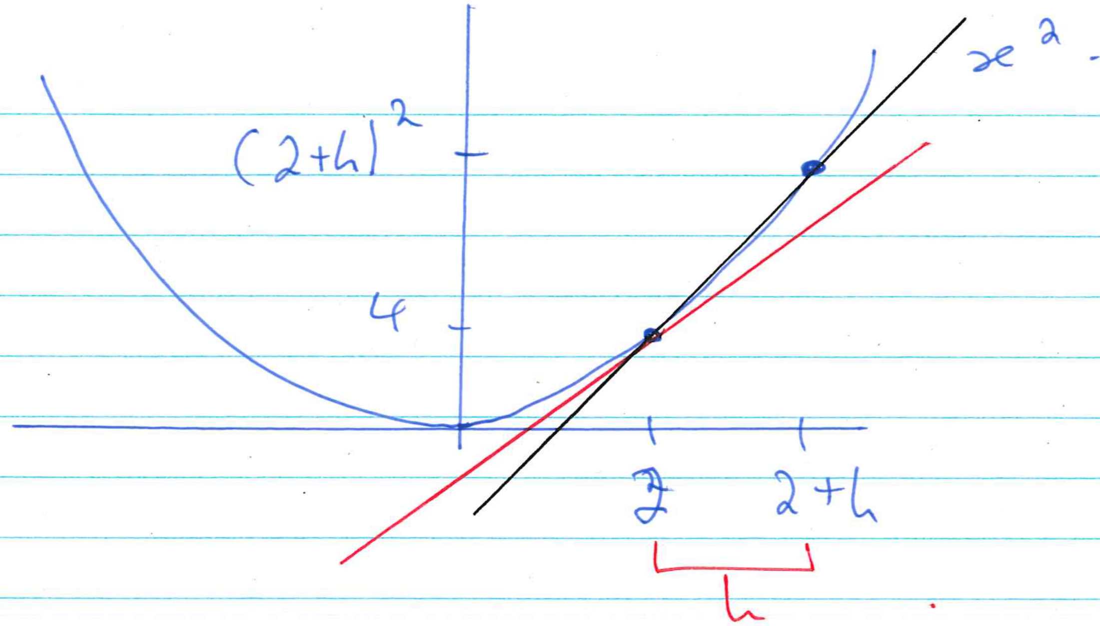


$$m_{sec} = \frac{(2+h)^2 - 4}{2+h - 2}$$

$$= \frac{(2+h)^2 - 4}{h}.$$

$$m_{tan} = \lim_{h\to 0} m_{sec} = \lim_{h\to 0} \frac{(2+h)^2 - 4}{h}.$$

Substitution will not work.

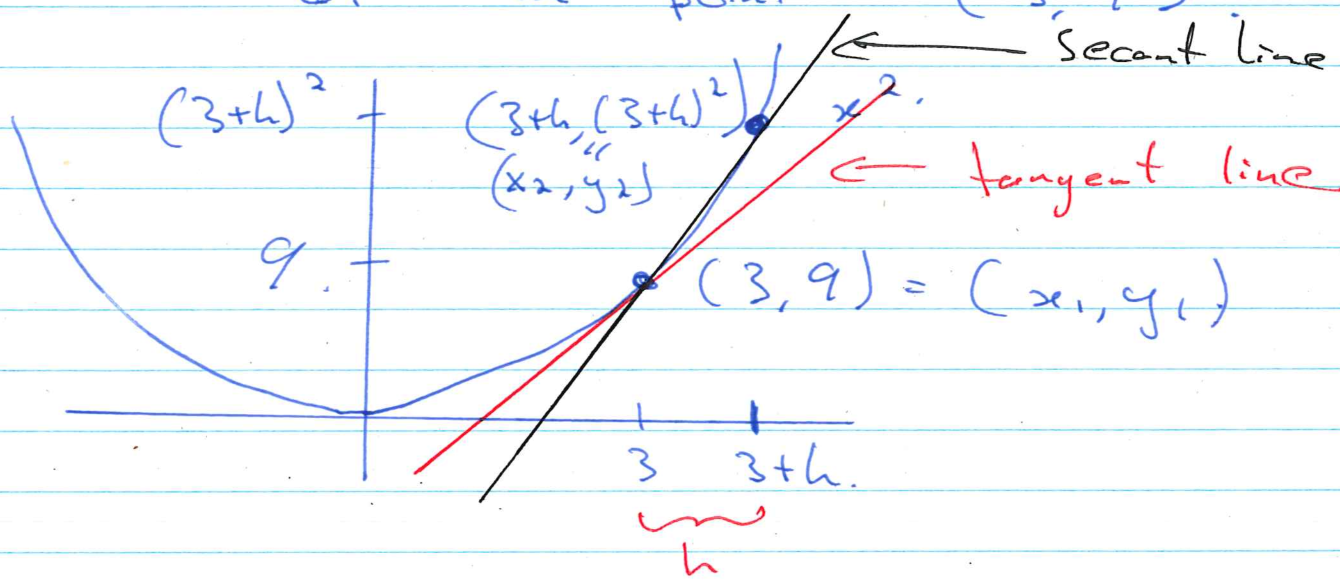
$$= \lim_{h\to 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h\to 0} \frac{4h + h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}}.$$

$$= \lim_{h \to 0} 4 + h.$$

$$= 4.$$

Example: Find the slope of the tangent line to x^2 at the point $(3, 9)$



$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(3+h)^2 - 9}{3+h-3}$$

$$= \frac{(3+h)^2 - 9}{h}.$$

$$m_{tan} = \lim_{h \to 0} m_{sec}$$

$$= \lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$$

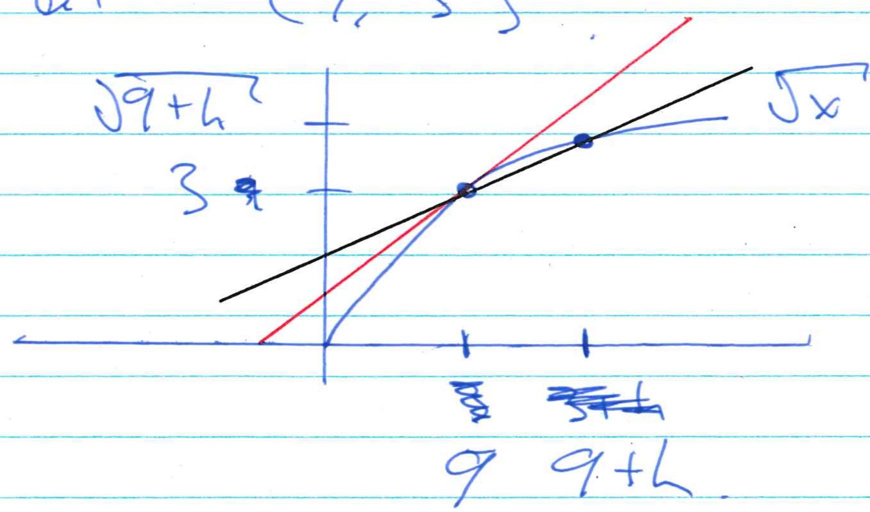
$$= \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 - \cancel{9}}{h}$$

$$= \lim_{h \to 0} \frac{6h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 6 + h = 6 + 0 = 6$$

Example: Find the slope of the tangent line to $f(x) = \sqrt{x}$ at $(9, 3)$



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$$m_{sec} = \frac{\sqrt{9+h} - 3}{9+h-9}$$

$$= \frac{\sqrt{9+h} - 3}{h}.$$

$$m_{tan} = \lim_{h\to 0} m_{sec}$$

$$= \lim_{h\to 0} \frac{\sqrt{9+h} - 3}{h} \cdot \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}.$$

$$= \lim_{h\to 0} \frac{9+h - 9}{h(\sqrt{9+h} + 3)}.$$

$$(\sqrt{9+h} - 3)(\sqrt{9+h} + 3)$$

$$= 9+h + \cancel{3\sqrt{9+h}} - \cancel{3\sqrt{9+h}} - 9$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)}.$$

$$= \lim_{h\to 0} \frac{1}{(\sqrt{9+h}+3)} = \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{3+3} = 1/6.$$