Sept. 23.

- Thursday Lab LIB room changed (logon) to M103.
- HW #1 Solutions posted Look at #5.
- HW #2 Due Monday
- Quiz #1 Friday
- Domain/Trig Worksheet posted.

Logarithms and Logarithmic Functions.

Let $2^y = 8$. mean the same thing.

We write $y = \log_2 8$.

(in this example $y = 3$ and we say $\log_2(8) = 3$).

In general,

if $x = b^y$ then $y = \log_b x$

Example: Find $y = \log_3(9)$.

$3^y = 9$ so $y = 2$.

Note, $\log_3(9) = \log_3(3^2) = 2$.

②

In general, $\log_b(b^x) = x$.

The function $g(x) = \log_b(x)$ is the inverse function of $f(x) = b^x$.

That is $g(f(x)) = \log_b(b^x) = x$.

input of x output of x.

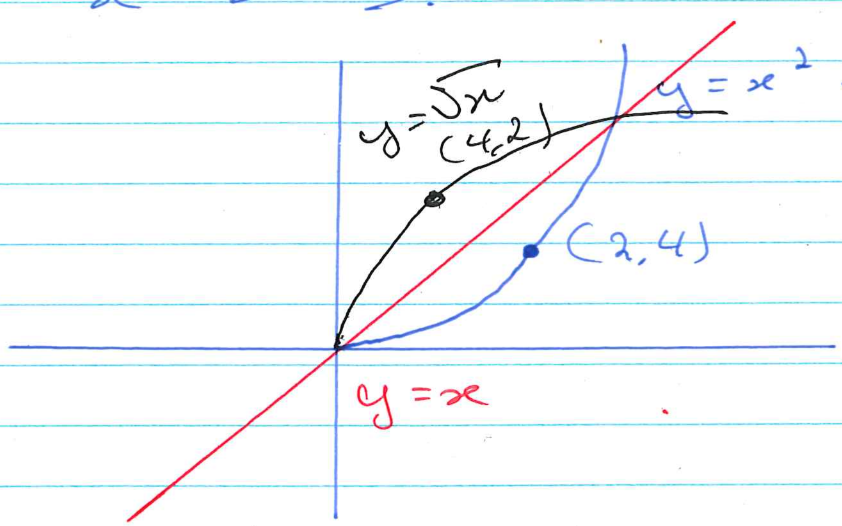
Do you know any other inverses?

- $\sin x$ $\sin^{-1} x = \arcsin x$.
- x^2 and $\sqrt{x}$?

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

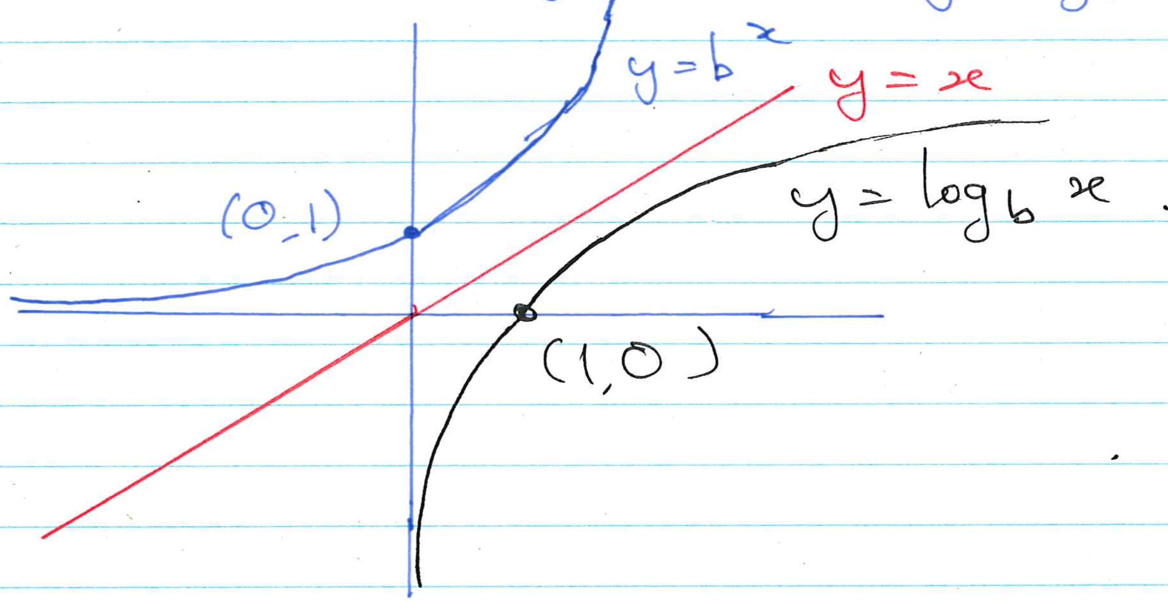
$x = \pm 3$.



③

the point $(2, 4)$ is on x^2.
the point $(4, 2)$ is on $\sqrt{x}$.

Now let's draw $y = b^x$ and $y = \log_b x$.



What is the domain of $f(x) = \log_b x$?

- $\{x \in \mathbb{R} : x > 0\}$

OR

- $(0, \infty)$.

What is the Domain of $f(x) = b^x$.

- $\{x \in \mathbb{R}\}$

What about the Range? of b^x.

- ~~$\{x \in \mathbb{R} : x > 0\}$~~
$\{y \in \mathbb{R} : y > 0\}$.

4

(aside:

$\{ \smile \in \mathbb{R} : \smile > 0 \}$.)

Especially important for us is

$$f(x) = \log_e x =. \ln x.$$

which is sometimes denoted $\ln x$. the so called natural logarithm.

Example: Solve for x:

$$\cancel{e^{x^2+1} = 5}. \quad (\ln 5 = \log_e 5.)$$

$$e^{x^2+1} = 5.$$

$$\ln(e^{x^2+1}) = \ln 5.$$

$$x^2 + 1 = \ln 5.$$

$$x^2 = \ln 5 - 1$$

$$x = \pm\sqrt{\ln 5 - 1}.$$

(aside:

$$\cancel{2^2 =} \; x^2 = 4$$

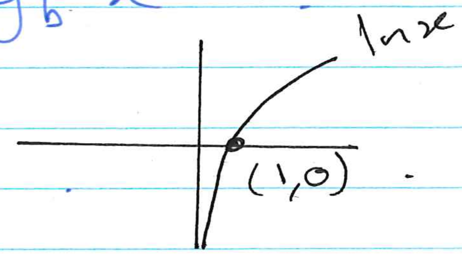
$$x = 2, -2.$$
)

To work with logarithms we need the following identities:

1) $\log_b(xy) = \log_b x + \log_b y$.

2) $\log_b(x/y) = \log_b x - \log_b y$.

3) $\log_b(x^p) = p \log_b x$.



Example: $\ln\left(\frac{1}{\sqrt{3x+1}}\right)$.

Simplify →

$$\ln\left(\frac{1}{\sqrt{3x+1}}\right) = \overset{0}{\ln(1)} - \ln\left(\sqrt{3x+1}\right).$$

using (2)

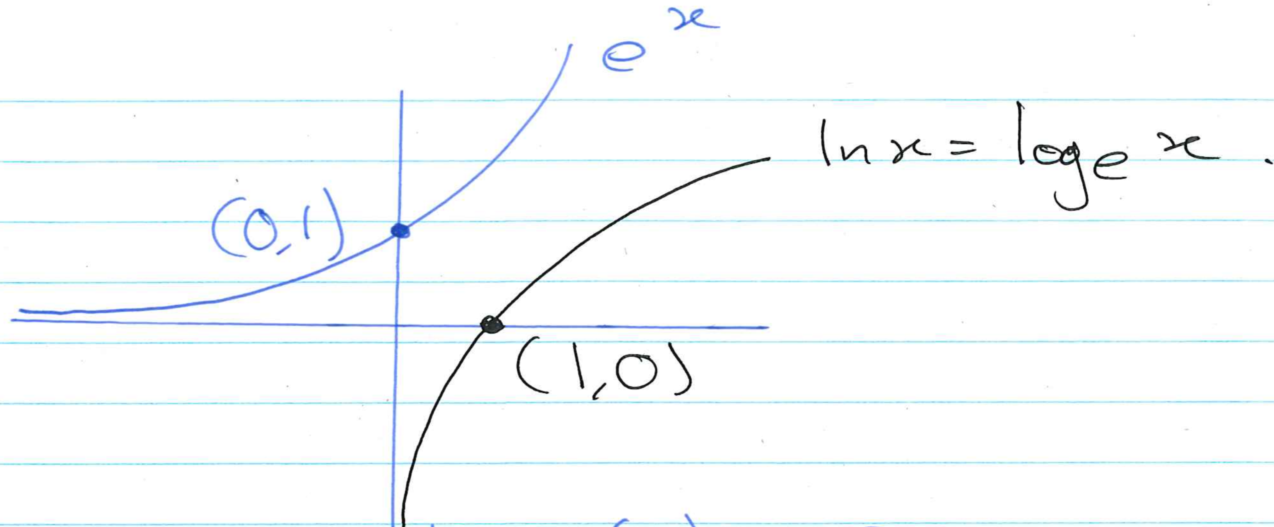
$$\left(\log_e\left(\frac{1}{\sqrt{3x+1}}\right) = \log_e(1) - \log_e\left(\sqrt{3x+1}\right) \right)$$

$$\ln\left(\frac{1}{\sqrt{3x+1}}\right) = \ln(1) - \ln\left(\sqrt{3x+1}\right)$$

$$= -\ln\left((3x+1)^{1/2}\right)$$

(3) → $= -\frac{1}{2}\ln(3x+1)$ //

⑥



In fact $\log_b(1) = 0$.

The three rules come from the three exponent laws.

1) comes from $b^x b^y = b^{x+y}$.

2) from $b^x / b^y = b^{x-y}$.

3) from $(b^x)^y = b^{xy}$.